# 9.4 Zero and Negative Exponents

**Essential Question** How can you define zero and negative exponents?

### 1 ACTIVITY: Finding Patterns and Writing Definitions

**Work with a partner.**

**a.** Talk about the following notation.

Thousands    Hundreds    Tens    Ones

$$4327 = 4 \cdot 10^3 + 3 \cdot 10^2 + 2 \cdot 10^1 + 7 \cdot 10^{\boxed{\phantom{0}}}$$

What patterns do you see in the first three exponents?
Continue the pattern to find the fourth exponent.
How would you define $10^0$? Explain.

**b.** Copy and complete the table.

| $n$ | 5 | 4 | 3 | 2 | 1 | 0 |
|-----|---|---|---|---|---|---|
| $2^n$ | | | | | | |

What patterns do you see in the first six values of $2^n$?
How would you define $2^0$? Explain.

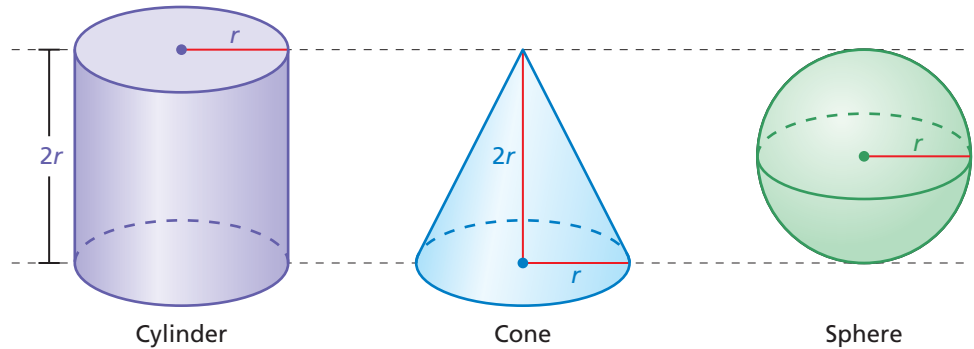**c.** Use the Quotient of Powers Property to complete the table.

$$\dfrac{3^5}{3^2} = 3^{5-2} = 3^3 \quad = 27$$

$$\dfrac{3^4}{3^2} = 3^{4-2} = \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\dfrac{3^3}{3^2} = 3^{3-2} = \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\dfrac{3^2}{3^2} = 3^{2-2} = \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

What patterns do you see in the first four rows of the table?
How would you define $3^0$? Explain.

## 2 ACTIVITY: Comparing Volumes

**Work with a partner.**

The quotients show three ratios of the volumes of the solids. Identify each ratio, find its value, and describe what it means.



Cylinder          Cone          Sphere

**a.** $2\pi r^3 \div \dfrac{2}{3}\pi r^3 = \boxed{\phantom{xx}}$

**b.** $\dfrac{4}{3}\pi r^3 \div \dfrac{2}{3}\pi r^3 = \boxed{\phantom{xx}}$

**c.** $2\pi r^3 \div \dfrac{4}{3}\pi r^3 = \boxed{\phantom{xx}}$

## 3 ACTIVITY: Writing a Definition

**Work with a partner.**

Compare the two methods used to simplify $\dfrac{3^2}{3^5}$. Then describe how you can rewrite a power with a negative exponent as a fraction.

*Method 1*

$$\frac{3^2}{3^5} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}} \cdot 3 \cdot 3 \cdot 3}$$

$$= \frac{1}{3^3}$$

*Method 2*

$$\frac{3^2}{3^5} = 3^{2-5}$$

$$= 3^{-3}$$

## What Is Your Answer?

**4. IN YOUR OWN WORDS** How can you define zero and negative exponents? Give two examples of each.

**Practice**

Use what you learned about zero and negative exponents to complete Exercises 5–8 on page 374.

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## 🔑 Key Ideas

**Zero Exponents**

**Words**  Any nonzero number to the zero power is equal to 1. Zero to the zero power, $0^0$, is *undefined*.

**Numbers**  $4^0 = 1$          **Algebra**  $a^0 = 1$, where $a \neq 0$

**Negative Exponents**

**Words**  For any integer $n$ and any number $a$ not equal to 0, $a^{-n}$ is equal to 1 divided by $a^n$.

**Numbers**  $4^{-2} = \dfrac{1}{4^2}$          **Algebra**  $a^{-n} = \dfrac{1}{a^n}$, where $a \neq 0$

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**EXAMPLE ❶  Evaluating Expressions**

**a.**  $3^{-4} = \dfrac{1}{3^4}$          Definition of negative exponent

$= \dfrac{1}{81}$          Evaluate power.

**b.**  $(-8.5)^{-4} \cdot (-8.5)^4 = (-8.5)^{-4+4}$          Add the exponents.

$= (-8.5)^0$          Simplify.

$= 1$          Definition of zero exponent

**c.**  $\dfrac{2^6}{2^8} = 2^{6-8}$          Subtract the exponents.

$= 2^{-2}$          Simplify.

$= \dfrac{1}{2^2}$          Definition of negative exponent

$= \dfrac{1}{4}$          Evaluate power.

## 🔴 On Your Own

**Now You're Ready**
Exercises 9–16

**Evaluate the expression.**

**1.**  $4^{-2}$          **2.**  $(-2)^{-5}$          **3.**  $6^{-8} \cdot 6^8$

**4.**  $\dfrac{(-3)^5}{(-3)^6}$          **5.**  $\dfrac{1}{5^7} \cdot \dfrac{1}{5^{-4}}$          **6.**  $\dfrac{4^5 \cdot 4^{-3}}{4^2}$

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**EXAMPLE** **2**    **Simplifying Expressions**

**a.**  $-5x^0 = -5(1)$      Definition of zero exponent

$= -5$      Multiply.

**b.**  $\dfrac{9y^{-3}}{y^5} = 9y^{-3-5}$      Subtract the exponents.

$= 9y^{-8}$      Simplify.

$= \dfrac{9}{y^8}$      Definition of negative exponent

**On Your Own**

**Simplify. Write the expression using only positive exponents.**

**7.**  $8x^{-2}$          **8.**  $b^0 \cdot b^{-10}$          **9.**  $\dfrac{z^6}{15z^9}$

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**EXAMPLE** **3**    **Real-Life Application**

**A drop of water leaks from a faucet every second. How many liters of water leak from the faucet in 1 hour?**

Convert 1 hour to seconds.

$$1\,\cancel{h} \times \frac{60\,\cancel{\text{min}}}{1\,\cancel{h}} \times \frac{60\,\text{sec}}{1\,\cancel{\text{min}}} = 3600\,\text{sec}$$

Water leaks from the faucet at a rate of $50^{-2}$ liter per second. Multiply the time by the rate.

$3600 \cdot 50^{-2} = 3600 \cdot \dfrac{1}{50^2}$      Definition of negative exponent

$= 3600 \cdot \dfrac{1}{2500}$      Evaluate power.

$= \dfrac{3600}{2500}$      Multiply.

$= 1\dfrac{11}{25} = 1.44$      Simplify.

Drop of water: $50^{-2}$ L

So, 1.44 liters of water leak from the faucet in 1 hour.

**On Your Own**

**10.** **WHAT IF?** In Example 4, the faucet leaks water at a rate of $5^{-5}$ liter per second. How many liters of water leak from the faucet in 1 hour?

## Vocabulary and Concept Check

1. **VOCABULARY** If $a$ is a nonzero number, does the value of $a^0$ depend on the value of $a$? Explain.

2. **WRITING** Explain how to evaluate $10^{-3}$.

3. **NUMBER SENSE** Without evaluating, order $5^0$, $5^4$, and $5^{-5}$ from least to greatest.

4. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Rewrite $\dfrac{1}{3 \cdot 3 \cdot 3}$ using a negative exponent.

Write 3 to the negative third power.

Write $\dfrac{1}{3}$ cubed as a power.

Write $(-3) \cdot (-3) \cdot (-3)$ as a power.

## Practice and Problem Solving

5. Use the Quotient of Powers Property to copy and complete the table.

6. What patterns do you see?

7. How would you define $5^0$? Why?

8. How can you rewrite $5^{-1}$ as a fraction?

| $n$ | 4 | 3 | 2 | 1 |
|---|---|---|---|---|
| $\dfrac{5^n}{5^2}$ | | | | |

**Evaluate the expression.**

**9.** $6^{-2}$

**10.** $158^0$

**11.** $\dfrac{4^3}{4^5}$

**12.** $\dfrac{-3}{(-3)^2}$

**13.** $(-2)^{-8} \cdot (-2)^8$

**14.** $3^{-3} \cdot 3^{-2}$

**15.** $\dfrac{1}{5^{-3}} \cdot \dfrac{1}{5^6}$

**16.** $\dfrac{(1.5)^2}{(1.5)^{-2} \cdot (1.5)^4}$

17. **ERROR ANALYSIS** Describe and correct the error in evaluating the expression.

✗
$(4)^{-3} = (-4)(-4)(-4)$
$= -64$

18. **SAND** The mass of a grain of sand is about $10^{-3}$ gram. About how many grains of sand are in the bag of sand?

SAND
10 kg

19. **CRITICAL THINKING** How can you write the number 1 as 2 to a power? 10 to a power?

**Simplify. Write the expression using only positive exponents.**

**②** **20.** $6y^{-4}$      **21.** $8^{-2} \cdot a^7$      **22.** $\dfrac{9c^3}{c^{-4}}$      **23.** $\dfrac{5b^{-2}}{b^{-3}}$

**24.** $\dfrac{8x^3}{2x^9}$      **25.** $3d^{-4} \cdot 4d^4$      **26.** $m^{-2} \cdot n^3$      **27.** $\dfrac{3^{-2} \cdot k^0 \cdot w^0}{w^{-6}}$

**METRIC UNITS** In Exercises 28–31, use the table.

**28.** How many millimeters are in a decimeter?

**29.** How many micrometers are in a centimeter?

**30.** How many nanometers are in a millimeter?

**31.** How many micrometers are in a meter?

| Unit of Length | Length |
|---|---|
| decimeter | $10^{-1}$ m |
| centimeter | $10^{-2}$ m |
| millimeter | $10^{-3}$ m |
| micrometer | $10^{-6}$ m |
| nanometer | $10^{-9}$ m |

**32. MICROBES** A species of bacteria is 10 micrometers long. A virus is 10,000 times smaller than the bacteria.

   **a.** Using the table above, find the length of the virus in meters.

   **b.** Is the answer to part (a) *less than*, *greater than*, or *equal to* one nanometer?

**33. BLOOD DONATION** Every 2 seconds, someone in the United States needs blood. A sample blood donation is shown. ($1 \text{ mm}^3 = 10^{-3}$ mL)

   **a.** One cubic millimeter of blood contains about $10^4$ white blood cells. How many white blood cells are in the donation? Write your answer in words.

   **b.** One cubic millimeter of blood contains about $5 \times 10^6$ red blood cells. How many red blood cells are in the donation? Write your answer in words.

   **c.** Compare your answers for parts (a) and (b).

**34. OPEN-ENDED** Write two different powers with negative exponents that have the same value.

**35.** **Reasoning** The rule for negative exponents states that $a^{-n} = \dfrac{1}{a^n}$. Explain why this rule does not apply when $a = 0$.

## Fair Game Review *What you learned in previous grades & lessons*

**Simplify the expression.**

**36.** $10^3 \cdot 10^6$        **37.** $10^2 \cdot 10$        **38.** $\dfrac{10^8}{10^4}$

**39. MULTIPLE CHOICE** Which data display best shows the variability of a data set? *Ž*

   **Ⓐ** bar graph               **Ⓑ** circle graph

   **Ⓒ** scatter plot            **Ⓓ** box-and-whisker plot