### 4.1 Domain and Range of a function

STATE STANDARDS

Essential Qusestion How can you find the domain and range of a function?

## (1) ACIIVIJY: The Domain and Range of a Function

Work with a partner. The table shows the number of adult and child tickets sold for a school concert.

| Input |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Nutput | Number of Adult Tickets, $\boldsymbol{x}$ Number of Child Tickets, $\boldsymbol{y}$

The variables $x$ and $y$ are related by the linear equation $4 x+2 y=16$.
a. Write the equation in function form by solving for $y$.
b. The domain of a function is the set of all input values. Find the domain of the function.
Domain =
$\square$
Why is $x=5$ not in the domain of the function?
Why is $x=\frac{1}{2}$ not in the domain of the function?
c. The range of a function is the set of all output values. Find the range of the function.

$$
\text { Range }=
$$

$\square$
d. Functions can be described in many ways.

- by an equation
- by an input-output table
- in words
- by a graph
- as a set of ordered pairs

Use the graph to write the function as a set of ordered pairs.



## 2 ACIIVITY: Finding Domains and Ranges

## Work with a partner.

- Copy and complete each input-output table.
- Find the domain and range of the function represented by the table.
a. $y=-3 x+4$

| $\boldsymbol{x}$ | -2 | -1 | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{y}$ |  |  |  |  |  |

b. $y=\frac{1}{2} x-6$

| $\boldsymbol{x}$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{y}$ |  |  |  |  |  |

c.


| $x$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ |  |  |  |  |  |

d.


| $x$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ |  |  |  |  |  |

## What is Your Answer?

3. IN YOUR OWN WORDS How can you find the domain and range of a function?
4. The following are general rules for finding a person's foot length.

To find the length $y$ (in inches) of a woman's foot, divide her shoe size $x$ by 3 and add 7 .

To find the length $y$ (in inches) of a man's

foot, divide his shoe size $x$ by 3 and add 7.3. © 2008 Zappos.com, Inc.
a. Write an equation for one of the statements.
b. Make an input-output table for the function in part (a). Use shoe sizes $5 \frac{1}{2}$ to 12 .
c. Label the domain and range of the function on the table.

## Practice

Use what you learned about the domain and range of a function to complete Exercise 3 on page 152.

Key Vocabulary
function, p. 150
domain, p. 150
range, p. 150
function form, p. 150


The ordered pair $(x, y)$ shows the output $y$ for an input $x$.

## Key Idea

## Functions

A function is a relationship that pairs each input with exactly one output. The domain is the set of all possible input values. The range is the set of all possible output values.


## EXAMPLE (1) Finding Domain and Range from a Graph



Find the domain and range of the function represented by the graph.

Write the ordered pairs. Identify the inputs and outputs.

$\therefore$ The domain is $-3,-1,1$, and 3 . The range is $-2,0,2$, and 4 .

## On Your Own

Now You're Ready
Exercises 4-6

Find the domain and range of the function represented by the graph.
1.

2.


An equation is in function form if it is solved for $y$.

$$
\begin{array}{cc}
x+y=1 & y=-x+1 \\
\text { not in function form } & \text { in function form }
\end{array}
$$

## EXAMPLE

## 2 Finding the Range of a function

| Input, $\boldsymbol{x}$ | $\mathbf{- 2 x}+\mathbf{8}$ | Output, $\boldsymbol{y}$ |
| :---: | :---: | :---: |
| -2 | $-2(-2)+8$ | 12 |
| 0 | $-2(0)+8$ | 8 |
| 2 | $-2(2)+8$ | 4 |
| 4 | $-2(4)+8$ | 0 |
| 6 | $-2(6)+8$ | -4 |

The domain of the function represented by $2 x+y=8$ is $-2,0,2,4$, and 6 . What is the range of the function represented by the table?
Write the function in function form.

$$
\begin{aligned}
2 x+y & =8 \\
y & =-2 x+8
\end{aligned}
$$

Use this form to make an input-output table.
$\therefore$ The range is $12,8,4,0$, and -4 .

## EXAMPLE 3 Real-Lffe Application

The table shows the percent $\boldsymbol{y}$ (in decimal form) of
 the moon that was visible at midnight $\boldsymbol{x}$ days after January 24, 2011. (a) Interpret the domain and range. (b) What percent of the moon was visible on January 26, 2011?
a. Zero days after January 24 is January 24 . One day

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 0 | 0.76 |
| 1 | 0.65 |
| 2 | 0.54 |
| 3 | 0.43 |
| 4 | 0.32 | after January 24 is January 25. So, the domain of $0,1,2,3$, and 4 represents January $24,25,26,27$, and 28.

The range is $0.76,0.65,0.54,0.43$, and 0.32 . These amounts are decreasing, so the moon was less visible each day.
b. January 26,2011 corresponds to the input $x=2$. When $x=2, y=0.54$. So, 0.54 , or $54 \%$ of the moon was visible on January 26, 2011.

## On Your Own

Now You're Ready
Exercises 9-11

Copy and complete the input-output table for the function. Then find the domain and range of the function represented by the table.
3. $y=2 x-3$
4. $x+y=-3$

| $\boldsymbol{x}$ | -1 | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{y}$ |  |  |  |  |


| $\boldsymbol{x}$ | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{y}$ |  |  |  |  |

5. The table shows the percent $y$ (in decimal form) of the moon that was visible at midnight $x$ days after December 17, 2012.

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |

(a) Interpret the domain and range.
(b) What percent of the moon was visible on December 21, 2012?

## Vocabulary and Concept Check

1. VOCABULARY Is the equation $2 x-3 y=4$ in function form? Explain.
2. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

Find the range of the function represented by the table.

Find the inputs of the function represented by the table.

Find the $x$-values of the function represented by $(2,7),(4,5)$, and $(6,-1)$.

Find the domain of the function represented by $(2,7),(4,5)$, and $(6,-1)$.

| $\boldsymbol{x}$ | 2 | 4 | 6 |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | 7 | 5 | -1 |

## Practice and Problem Solving

3. The number of earrings and headbands you can buy with $\$ 24$ is represented by the equation $8 x+4 y=24$. The table shows the number of earrings and headbands.
a. Write the equation in function form.
b. Find the domain and range.
c. Why is $x=6$ not in the domain of the function?

| Earrings, $\boldsymbol{x}$ | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| Headbands, $\boldsymbol{y}$ | 6 | 4 | 2 | 0 |

Find the domain and range of the function represented by the graph.
(1)
4.

5.

6.

7. ERROR ANALYSIS Describe and correct the error in finding the domain and range of the function represented by the graph.
8. REASONING Find the domain and range of the function represented by the table.

| Tickets, $\boldsymbol{x}$ | 2 | 3 | 5 | 8 |
| :--- | :---: | :---: | :---: | :---: |
| Cost, $\boldsymbol{y}$ | $\$ 14$ | $\$ 21$ | $\$ 35$ | $\$ 56$ |

Copy and complete the input-output table for the function. Then find the domain and range of the function represented by the table.
9. $y=6 x+2$

| $\boldsymbol{x}$ | -1 | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{y}$ |  |  |  |  |

10. $y=-\frac{1}{4} x-2$

| $\boldsymbol{x}$ | 0 | 4 | 8 | 12 |
| :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{y}$ |  |  |  |  |

11. $y=1.5 x+3$

| $\boldsymbol{x}$ | -1 | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{y}$ |  |  |  |  |

12. VAULTING In the sport of vaulting, a vaulter performs a routine while on a moving horse. For each round $x$ of competition, the vaulter receives a score $y$ from 1 to 10 .
a. Find the domain and range of the function represented by the table.
b. Interpret the domain and range.
c. What is the mean score of the vaulter?
13. MANATEE Florida's state marine mammal is the manatee. A manatee eats about $12 \%$ of its body weight each day.
a. Write an equation in function form that represents the amount $y$ (in pounds) of food a manatee eats each day for its weight $x$.
b. Create an input-output table for the equation in part (a). Use the inputs $150,300,450,600,750$, and 900.

c. Find the domain and range of the function represented by the table.

d. An aquatic center has manatees that weigh 300 pounds, 750 pounds, and 1050 pounds. How many pounds of food do all three manatees eat in a day? in a week?
14. Thinking Describe the domain and range of the function.
a. $y=|x|$
b. $y=-|x|$
c. $y=|x|-6$
d. $y=-|x|+4$

## Fair Game Review what you learned in previous grades \& lessons

Graph the linear equation. SECTION 2.1
15. $y=2 x+8$
16. $5 x+6 y=12$
17. $-x-3 y=2$
18. $y=7 x-5$
19. MULTIPLE CHOICE The minimum number of people needed for a group rate at an amusement park is 8 . Which inequality represents the number of people needed to get the group rate?

SKILLS REVIEW HANDBOOK
(A) $x \leq 8$
(B) $x>8$
(C) $x<8$
(D) $x \geq 8$

