

6.4 Surface Areas of Pyramids

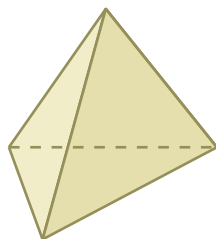


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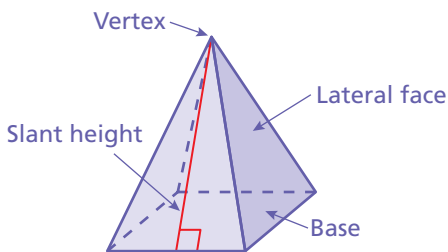
MA.7.G.2.1

Essential Question How can you find the surface area of a pyramid?

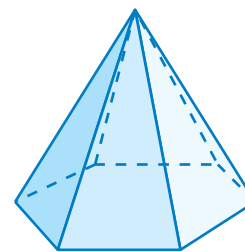
Even though many well-known **pyramids** have square bases, the base of a pyramid can be any polygon.



Triangular Base



Square Base



Hexagonal Base

1 ACTIVITY: Making a Scale Model

Work with a partner. Each pyramid has a square base.

- Draw a net for a scale model of one of the pyramids. Describe your scale.
- Cut out the net and fold it to form a pyramid.
- Find the lateral surface area of the real-life pyramid.

a. Cheops Pyramid in Egypt



Side = 230 m, Slant height \approx 186 m

b. Muttart Conservatory in Edmonton



Side = 26 m, Slant height \approx 27 m

c. Louvre Pyramid in Paris



Side = 35 m, Slant height \approx 28 m

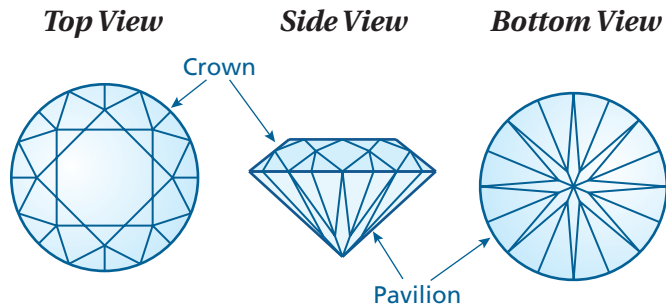
d. Pyramid of Caius Cestius in Rome



Side = 22 m, Slant height \approx 29 m

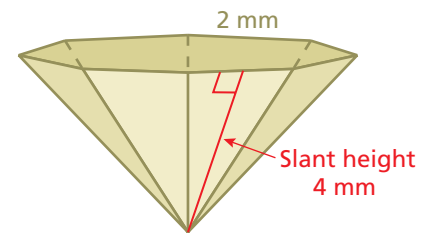
2 ACTIVITY: Estimation

Work with a partner. There are many different types of gemstone cuts. Here is one called a brilliant cut.



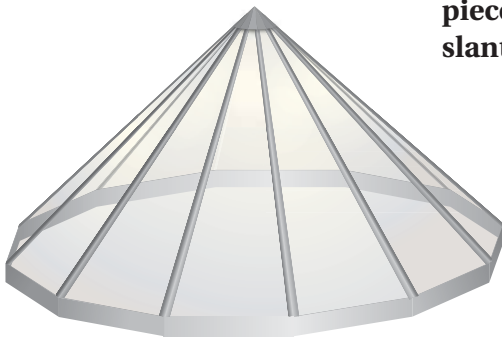
The size and shape of the pavilion can be approximated by an octagonal pyramid.

- What does octagonal mean?
- Draw a net for the pyramid.
- Find the lateral surface area of the pyramid.



3 ACTIVITY: Building a Skylight

Work with a partner. The skylight has 12 triangular pieces of glass. Each piece has a base of 1 foot and a slant height of 3 feet.



- How much glass will you need to make the skylight?
- Can you cut the 12 glass triangles from a sheet of glass that is 4 feet by 8 feet? If so, draw a diagram showing how this can be done.

What Is Your Answer?

- IN YOUR OWN WORDS** How can you find the surface area of a pyramid? Draw a diagram with your explanation.

Practice

Use what you learned about the surface area of a pyramid to complete Exercises 4–6 on page 274.

Key Vocabulary

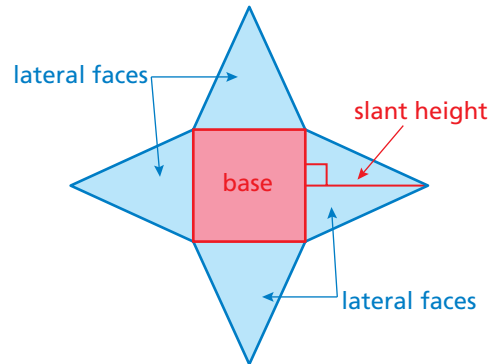
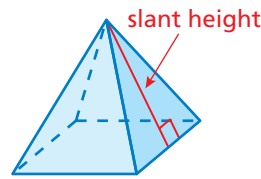
regular pyramid,
p. 272
slant height, p. 272

A **regular pyramid** is a pyramid whose base is a regular polygon. The lateral faces are triangles. The height of each triangle is the **slant height** of the pyramid.

Key Idea

Surface Area of a Pyramid

The surface area S of a pyramid is the sum of the areas of the base and the lateral faces.

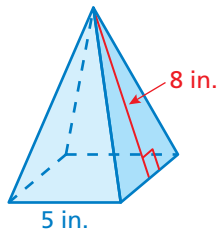


$$S = \text{area of base} + \text{areas of lateral faces}$$

Remember

In a regular polygon, all of the sides have the same length and all of the angles have the same measure.

EXAMPLE 1 Finding the Surface Area of a Square Pyramid



Find the surface area of the regular pyramid.

Draw a net.

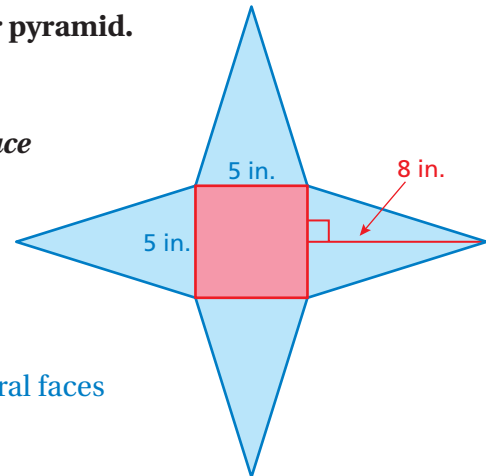
Area of base *Area of a lateral face*

$$5 \cdot 5 = 25 \qquad \frac{1}{2} \cdot 5 \cdot 8 = 20$$

Find the sum of the areas of the base and the lateral faces.

$$\begin{aligned} S &= \text{area of base} + \text{areas of lateral faces} \\ &= 25 + 20 + 20 + 20 + 20 \\ &= 105 \end{aligned}$$

There are 4 identical lateral faces. Count the area 4 times.

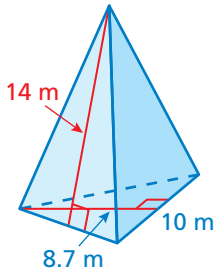


∴ The surface area is 105 square inches.

On Your Own

1. What is the surface area of a square pyramid with a base side length of 9 centimeters and a slant height of 7 centimeters?

EXAMPLE 2 Finding the Surface Area of a Triangular Pyramid



Find the surface area of the regular pyramid.

Draw a net.

Area of base

$$\frac{1}{2} \cdot 10 \cdot 8.7 = 43.5$$

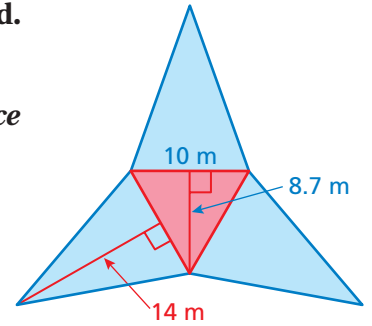
Area of a lateral face

$$\frac{1}{2} \cdot 10 \cdot 14 = 70$$

Find the sum of the areas of the base and the lateral faces.

$$\begin{aligned} S &= \text{area of base} + \text{areas of lateral faces} \\ &= 43.5 + \underbrace{70 + 70 + 70} \\ &= 253.5 \end{aligned}$$

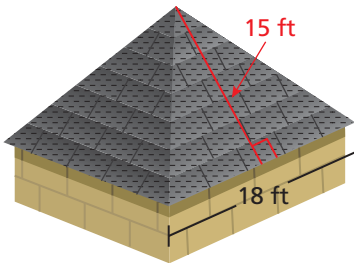
There are 3 identical lateral faces. Count the area 3 times.



∴ The surface area is 253.5 square meters.

EXAMPLE 3 Real-Life Application

A roof is shaped like a square pyramid. One bundle of shingles covers 25 square feet. How many bundles should you buy to cover the roof?



The base of the roof does not need shingles. So, find the sum of the areas of the lateral faces of the pyramid.

Area of a lateral face

$$\frac{1}{2} \cdot 18 \cdot 15 = 135$$

There are four identical lateral faces. So, the sum of the areas of the lateral faces is

$$135 + 135 + 135 + 135 = 540.$$

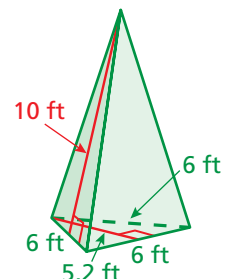
Because one bundle of shingles covers 25 square feet, it will take $540 \div 25 = 21.6$ bundles to cover the roof.

∴ So, you should buy 22 bundles of shingles.

On Your Own

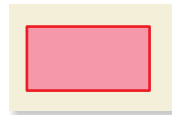
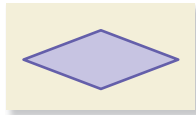
Now You're Ready
Exercises 4–12

- What is the surface area of the pyramid at the right?
- WHAT IF?** In Example 3, one bundle of shingles covers 32 square feet. How many bundles should you buy to cover the roof?



Vocabulary and Concept Check

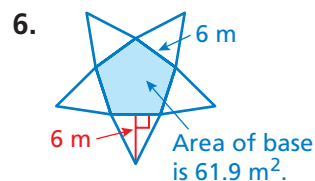
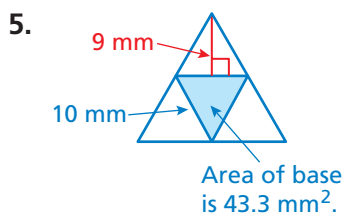
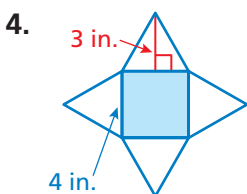
1. **VOCABULARY** Which of the polygons could be the base for a regular pyramid?



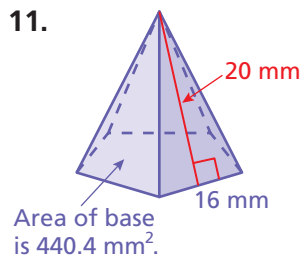
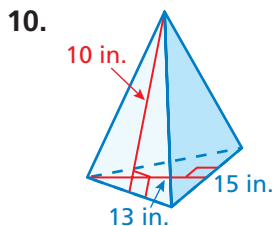
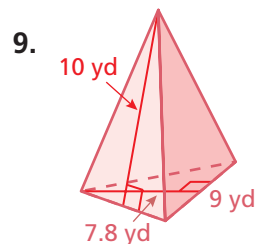
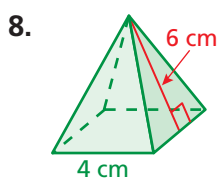
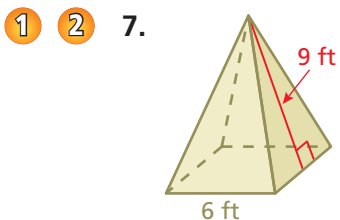
2. **VOCABULARY** Can a pyramid have rectangles as lateral faces? Explain.
3. **CRITICAL THINKING** Why is it helpful to know the slant height of a pyramid to find its surface area?

Practice and Problem Solving

Use the net to find the surface area of the regular pyramid.

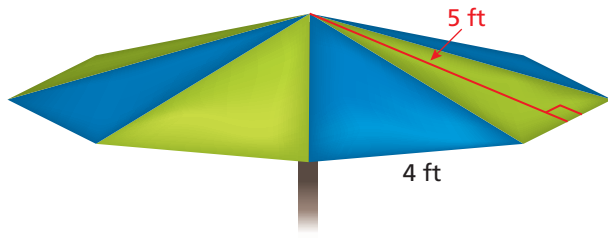
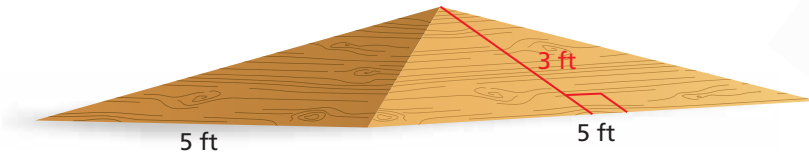


In Exercises 7–11, find the surface area of the regular pyramid.



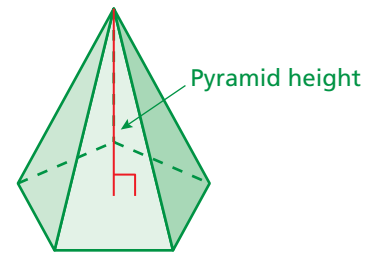
- 3 12. **LAMPSHADE** The base of the lampshade is a regular hexagon with a side length of 8 inches. Estimate the amount of glass needed to make the lampshade.
13. **GEOMETRY** The surface area of a square pyramid is 85 square meters. The base length is 5 meters. What is the slant height?

14. **BMX** You are building a bike ramp that is shaped like a square pyramid. You use two 4-foot by 8-foot sheets of plywood. How much plywood do you have left over?



15. **UMBRELLA** You are making an umbrella that is shaped like a regular octagonal pyramid.
- Estimate the amount of fabric that is needed to make the umbrella.
 - The fabric comes in rolls that are 72 inches wide. You don't want to cut the fabric "on the bias". Find out what this means. Then, draw a diagram of how you can cut the fabric most efficiently.
 - How much fabric is wasted?

16. **REASONING** The *height* of a pyramid is the distance between the base and the top of the pyramid. Which is greater, the height of a pyramid or the slant height? Explain your reasoning.

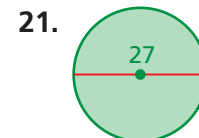
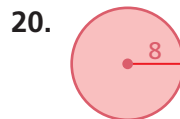
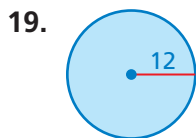


17. **TETRAHEDRON** A tetrahedron is a triangular pyramid whose four faces are identical equilateral triangles. The total lateral surface area is 93 square centimeters. Find the surface area of the tetrahedron.
18. **Reasoning** Is the total area of the lateral faces of a pyramid *greater than*, *less than*, or *equal* to the area of the base? Explain.



Fair Game Review what you learned in previous grades & lessons

Find the area and circumference of the circle. Use 3.14 for π .



22. **MULTIPLE CHOICE** A youth baseball diamond is similar to a professional baseball diamond. The ratio of the perimeters is 2 : 3. The distance between bases on a youth diamond is 60 feet. What is the distance between bases on a professional diamond?

(A) 40 ft (B) 90 ft (C) 120 ft (D) 180 ft