

2.3 Multiplying Fractions



STATE STANDARDS

- MA.6.A.1.1
- MA.6.A.1.2
- MA.6.A.1.3
- MA.6.A.5.3

Essential Question What does it mean to multiply fractions?

1 EXAMPLE: Multiplying Fractions

A bottle of water is $\frac{4}{5}$ full. You drink $\frac{2}{3}$ of the water. How much do you drink?



THINK ABOUT THE QUESTION: To help you think about this question, rewrite the question.

Words: What is $\frac{2}{3}$ of $\frac{4}{5}$?

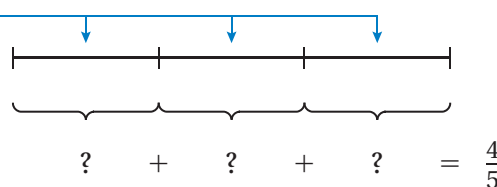
Numbers: $\frac{2}{3} \times \frac{4}{5} = ?$

Here is one way to get the answer.

- Draw a length of $\frac{4}{5}$.



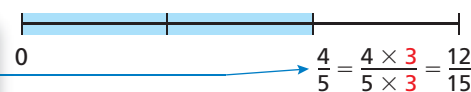
Because you want to find $\frac{2}{3}$ of the length, divide it into 3 equal sections.



Now, you need to think of a way to divide $\frac{4}{5}$ into three equal parts.

- Rewrite $\frac{4}{5}$ as a fraction whose numerator is divisible by 3.

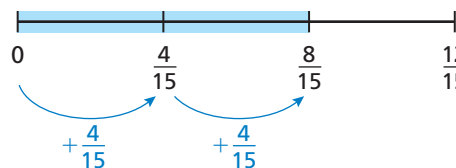
Because the length is divided into 3 equal sections, multiply the numerator and denominator by 3.



In this form, you see that $\frac{12}{15}$ can be divided into three equal parts of $\frac{4}{5}$.

- Each part is $\frac{4}{15}$ of the water and you drank two of them. Written as multiplication, you have

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

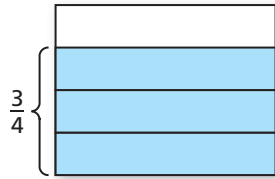
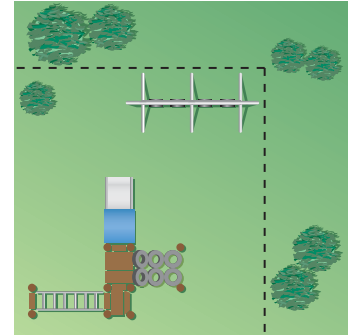


- So, you drank $\frac{8}{15}$ of the water.

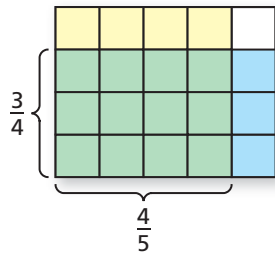
2 EXAMPLE: Multiplying Fractions

A park has a playground that is $\frac{3}{4}$ of its width and $\frac{4}{5}$ of its length.

What fraction of the park is covered by the playground?



Fold a piece of paper horizontally into fourths and shade three of the fourths to represent $\frac{3}{4}$.



Fold the paper vertically into fifths and shade $\frac{4}{5}$ of the paper another color.

Count the total number of squares. This number is the denominator. The numerator is the number of squares shaded with both colors.

∴ $\frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}$. So, $\frac{3}{5}$ of the park is covered by the playground.

Inductive Reasoning

Work with a partner. Complete the table using a model or paper folding.

Exercise	Verbal Expression	Answer
1. $\frac{2}{3} \times \frac{4}{5}$	$\frac{2}{3}$ of $\frac{4}{5}$	$\frac{8}{15}$
2. $\frac{3}{4} \times \frac{4}{5}$	$\frac{3}{4}$ of $\frac{4}{5}$	$\frac{3}{5}$
5. $\frac{2}{3} \times \frac{5}{6}$		
6. $\frac{1}{6} \times \frac{1}{4}$		
7. $\frac{2}{5} \times \frac{1}{2}$		
8. $\frac{5}{8} \times \frac{4}{5}$		

What Is Your Answer?

- IN YOUR OWN WORDS** What does it mean to multiply fractions?
- Write a general rule for multiplying fractions.

Practice

Use what you learned about multiplying fractions to complete Exercises 5–12 on page 60.

Key Idea
Multiplying Fractions**Words** Multiply the numerators and multiply the denominators.

Numbers $\frac{3}{7} \times \frac{1}{2} = \frac{3 \times 1}{7 \times 2} = \frac{3}{14}$

Algebra $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$, where $b, d \neq 0$

EXAMPLE 1 Multiplying Fractions

Find $\frac{1}{5} \times \frac{1}{3}$.

$$\frac{1}{5} \times \frac{1}{3} = \frac{1 \times 1}{5 \times 3}$$

Multiply the numerators.

Multiply the denominators.

$$= \frac{1}{15}$$

Simplify.

EXAMPLE 2 Multiplying Fractions with Common Factors

Find $\frac{8}{9} \times \frac{3}{4}$.

Estimate $1 \times \frac{1}{2} = \frac{1}{2}$

$$\frac{8}{9} \times \frac{3}{4} = \frac{8 \times 3}{9 \times 4}$$

Multiply the numerators.

Multiply the denominators.

$$= \frac{\overset{2}{\cancel{8}} \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times \underset{1}{\cancel{4}}}$$

Divide out common factors.

$$= \frac{2}{3}$$

Simplify.

So, the product is $\frac{2}{3}$.

Reasonable? $\frac{2}{3} \approx \frac{1}{2}$ ✓

Study Tip

When the numerator of one fraction is the same as the denominator of another fraction, you can use mental math to multiply. For example, $\frac{4}{5} \times \frac{5}{9} = \frac{4}{9}$ because you can divide out the common factor 5.

On Your Own

Multiply. Write the answer in simplest form.

1. $\frac{1}{2} \times \frac{5}{6}$

2. $\frac{7}{8} \times \frac{1}{4}$

3. $\frac{3}{7} \times \frac{2}{3}$

4. $\frac{4}{9} \times \frac{3}{10}$

Now You're Ready
Exercises 5–20

EXAMPLE 3 Standardized Test Practice

What is the value of $p \cdot \frac{7}{8} - q$ when $p = \frac{4}{5}$ and $q = \frac{1}{4}$?

- (A) $\frac{1}{4}$ (B) $\frac{9}{20}$ (C) $\frac{1}{2}$ (D) 1

$$p \cdot \frac{7}{8} - q = \frac{4}{5} \cdot \frac{7}{8} - \frac{1}{4}$$

Substitute $\frac{4}{5}$ for p and $\frac{1}{4}$ for q .

$$= \frac{\overset{1}{\cancel{4}} \cdot 7}{5 \cdot \underset{2}{\cancel{8}}} - \frac{1}{4}$$

Multiply. Divide out the common factor 4.

$$= \frac{7}{10} - \frac{1}{4}$$

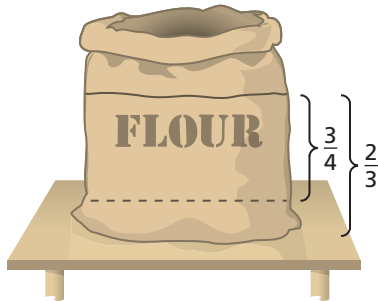
Simplify.

$$= \frac{14}{20} - \frac{5}{20} = \frac{9}{20}$$

Subtract.

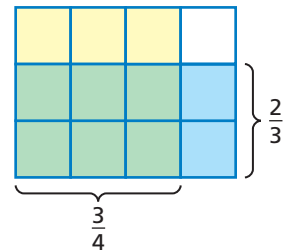
∴ The correct answer is (B).

EXAMPLE 4 Real-Life Application



You have $\frac{2}{3}$ of a bag of flour. You use $\frac{3}{4}$ of the flour to make empanada dough. How much of the entire bag do you use to make the dough?

Method 1: Use a model. Six of the 12 squares have both types of shading.



∴ So, you use $\frac{6}{12} = \frac{1}{2}$ of the entire bag.

Method 2: To find $\frac{3}{4}$ of $\frac{2}{3}$, multiply.

$$\frac{3}{4} \times \frac{2}{3} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \times \underset{1}{\cancel{3}}} = \frac{1}{2}$$

Multiply the numerators and the denominators.
Divide out common factors.

Simplify.

∴ So, you use $\frac{1}{2}$ of the entire bag.

On Your Own

Now You're Ready
Exercises 24–31

5. Evaluate $a + b \cdot \frac{1}{12}$ when $a = \frac{5}{6}$ and $b = \frac{2}{3}$.

6. **WHAT IF?** In Example 4, you use $\frac{1}{4}$ of the flour to make the dough. How much of the entire bag do you use to make the dough?

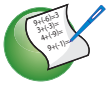


Vocabulary and Concept Check

- WRITING** Explain how to multiply two fractions.
- OPEN-ENDED** Give three different sets of two fractions each having the same product.
- REASONING** Name the missing denominator.

$$\frac{3}{7} \times \frac{1}{\square} = \frac{3}{28}$$

- NUMBER SENSE** Is $\frac{2}{3} \times \frac{5}{8}$ the same as $\frac{5}{8} \times \frac{2}{3}$? Explain.



Practice and Problem Solving

Multiply. Write the answer in simplest form.

- | | | | | | |
|---|---|--------------------------------------|--------------------------------------|--|--|
| 1 | 2 | 5. $\frac{1}{7} \times \frac{2}{3}$ | 6. $\frac{5}{8} \times \frac{1}{2}$ | 7. $\frac{1}{4} \times \frac{2}{5}$ | 8. $\frac{3}{7} \times \frac{1}{4}$ |
| | | 9. $\frac{2}{3} \times \frac{4}{5}$ | 10. $\frac{5}{7} \times \frac{7}{8}$ | 11. $\frac{3}{8} \times \frac{1}{9}$ | 12. $\frac{5}{6} \times \frac{2}{5}$ |
| | | 13. $\frac{5}{12} \times 10$ | 14. $6 \times \frac{7}{8}$ | 15. $\frac{3}{4} \times \frac{8}{15}$ | 16. $\frac{4}{9} \times \frac{4}{5}$ |
| | | 17. $\frac{3}{7} \times \frac{3}{7}$ | 18. $\frac{5}{6} \times \frac{2}{9}$ | 19. $\frac{13}{18} \times \frac{6}{7}$ | 20. $\frac{7}{9} \times \frac{21}{10}$ |

- ERROR ANALYSIS** Describe and correct the error in finding the product.

$$\times \quad \frac{2}{5} \times \frac{3}{10} = \frac{4}{10} \times \frac{3}{10} = \frac{4 \times 3}{10} = \frac{12}{10} = 1\frac{1}{5}$$

- AQUARIUM** In an aquarium, $\frac{2}{5}$ of the fish are surgeonfish. Of these, $\frac{3}{4}$ are yellow tangs. What fraction of all fish in the aquarium are yellow tangs?
- JUMP ROPE** You exercise for $\frac{3}{4}$ of an hour. You jump rope for $\frac{1}{3}$ of that time. What fraction of the hour do you spend jumping rope?

ALGEBRA Evaluate the expression when $a = \frac{3}{4}$, $b = \frac{1}{6}$, and $c = \frac{2}{5}$.

- | | | | | |
|---|----------------------------------|-------------------------------------|------------------------|-----------------|
| 3 | 24. $a \cdot \frac{5}{12}$ | 25. $\frac{4}{7} \cdot b$ | 26. $a \cdot b$ | 27. $c \cdot a$ |
| | 28. $\frac{5}{9}a + \frac{1}{9}$ | 29. $\frac{14}{15} - \frac{7}{12}c$ | 30. $bc + \frac{2}{3}$ | 31. $ab + c$ |

Multiply. Write the answer in simplest form.

32. $\frac{1}{2} \times \frac{3}{5} \times \frac{4}{9}$

33. $\frac{3}{4} \times \frac{5}{8} \times \frac{6}{25}$

34. $\frac{4}{7} \times \frac{2}{3} \times \frac{9}{16}$

35. $\frac{5}{6} \times \frac{4}{15} \times \frac{7}{10}$

36. $\left(\frac{9}{10}\right)^2$


37. $\left(\frac{3}{5}\right)^3$


38. $\left(\frac{4}{5}\right)^2 \times \left(\frac{3}{4}\right)^2$

39. $\left(\frac{5}{6}\right)^2 \times \left(\frac{3}{7}\right)^2$

Without finding the product, copy and complete the statement using <, >, or =.

Explain your reasoning.

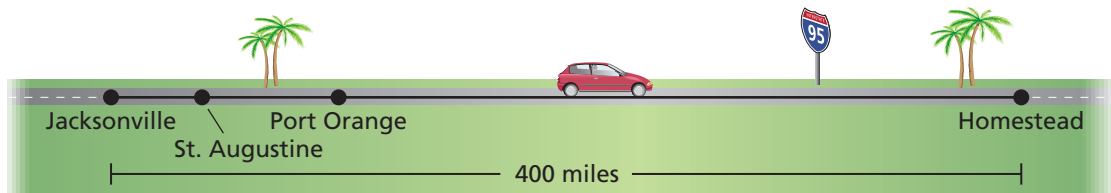
40. $\frac{4}{7}$  $\left(\frac{9}{10} \times \frac{4}{7}\right)$

41. $\left(\frac{5}{8} \times \frac{22}{15}\right)$  $\frac{5}{8}$

42. $\frac{5}{6}$  $\left(\frac{5}{6} \times \frac{7}{7}\right)$

43. **OPEN-ENDED** Find a fraction that when multiplied by $\frac{1}{2}$ is less than $\frac{1}{4}$.

44. **DISTANCES** You're traveling from Jacksonville to Homestead on Interstate 95. When you get to St. Augustine, you are $\frac{2}{5}$ the distance to Port Orange. When you get to Port Orange, you are $\frac{1}{4}$ the distance to Homestead. What is the distance from Jacksonville to St. Augustine?



45. **PETS** You ask 150 people about their pets. The results show that $\frac{9}{25}$ of the people own a dog. Of the people that own a dog, $\frac{1}{6}$ of them also own a cat.
- What fraction of the people own a dog and a cat?
 - Reasoning** How many people own a dog, but not a cat? Explain.



Fair Game Review What you learned in previous grades & lessons

Write the mixed number as an improper fraction.

46. $9\frac{1}{3}$

47. $4\frac{3}{8}$

48. $7\frac{3}{4}$

49. $3\frac{5}{6}$

50. **MULTIPLE CHOICE** A science experiment calls for $\frac{3}{4}$ cup of baking powder. You have $\frac{1}{3}$ cup of baking powder. How much more baking powder do you need?

(A) $\frac{1}{4}$ cup

(B) $\frac{5}{12}$ cup

(C) $\frac{4}{7}$ cup

(D) $1\frac{1}{12}$ cups