Name ____ Date __________

11.4 The Sine and Cosine Ratios

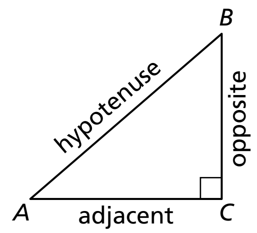
For use with Exploration 11.4

Essential Question How is a right triangle used to find the sine and cosine of an acute angle? Is there a unique right triangle that must be used?

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The *sine* of $\angle A$ and *cosine* of $\angle A$ (written as $\sin A$ and $\cos A$, respectively) are defined as follows.

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

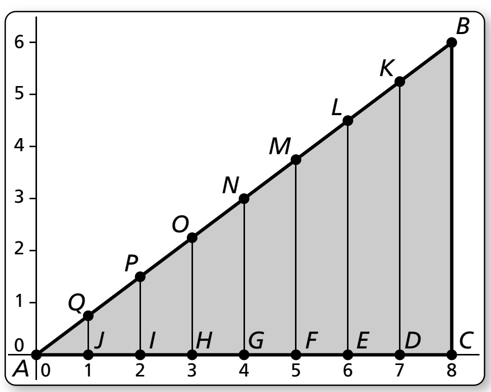


1 EXPLORATION: Calculating Sine and Cosine Ratios

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct $\triangle ABC$, as shown. Construct segments perpendicular to $\overline{AC}$ to form right triangles that share vertex A and are similar to $\triangle ABC$ with vertices, as shown.



Sample
Points
$A(0, 0)$
$B(8, 6)$
$C(8, 0)$
Angle
$m\angle BAC = 36.87°$

Name____ Date__________

11.4 The Sine and Cosine Ratios (continued)

1 EXPLORATION: Calculating Sine and Cosine Ratios (continued)

b. Calculate each given ratio to complete the table for the decimal values of sin A and cos A for each right triangle. What can you conclude?

Sine ratio	$\frac{BC}{AB}$	$\frac{KD}{AK}$	$\frac{LE}{AL}$	$\frac{MF}{AM}$	$\frac{NG}{AN}$	$\frac{OH}{AO}$	$\frac{PI}{AP}$	$\frac{QJ}{AQ}$
sin *A*								
Cosine ratio	$\frac{AC}{AB}$	$\frac{AD}{AK}$	$\frac{AE}{AL}$	$\frac{AF}{AM}$	$\frac{AG}{AN}$	$\frac{AH}{AO}$	$\frac{AI}{AP}$	$\frac{AJ}{AQ}$
cos *A*								

Communicate Your Answer

2. How is a right triangle used to find the sine and cosine of an acute angle? Is there a unique right triangle that must be used?

3. In Exploration 1, what is the relationship between $\angle A$ and $\angle B$ in terms of their measures? Find sin B and cos B. How are these two values related to sin A and cos A? Explain why these relationships exist.

Name ____ Date __________

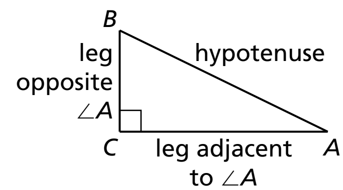
11.4 Practice

For use after Lesson 11.4

Core Concepts

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written as $\sin A$ and $\cos A$) are defined as follows.



$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

Notes:

Sine and Cosine of Complementary Angles

The sine of an acute angle is equal to the cosine of its complement. The cosine of an acute angle is equal to the sine of its complement.

Let A and B be complementary angles. Then the following statements are true.

$$\sin A = \cos(90^\circ - A) = \cos B \qquad \sin B = \cos(90^\circ - B) = \cos A$$

$$\cos A = \sin(90^\circ - A) = \sin B \qquad \cos B = \sin(90^\circ - B) = \sin A$$

Notes:

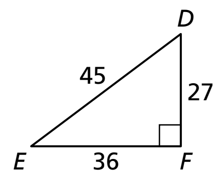
Name______________________________ Date__________

Worked-Out Examples

Example #1

Find sin D, sin E, cos D, and cos E. Write each answer as a fraction and as a decimal rounded to four places.

$$\sin D = \frac{\text{opp}}{\text{hyp}} = \frac{36}{45} = \frac{4}{5} = 0.8000$$
$$\cos D = \frac{\text{adj}}{\text{hyp}} = \frac{27}{45} = \frac{3}{5} = 0.6000$$
$$\sin E = \frac{\text{opp}}{\text{hyp}} = \frac{27}{45} = \frac{3}{5} = 0.6000$$
$$\cos E = \frac{\text{adj}}{\text{hyp}} = \frac{36}{45} = \frac{4}{5} = 0.8000$$



Example #2

Write the expression in terms of cosine.

$\sin 37° = \cos(90° - 37°) = \cos 53°$

Example #3

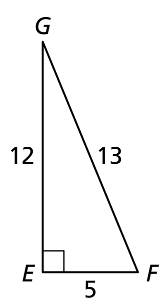
Write the expression in terms of sine.

$\cos 59° = \sin(90° - 59°) = \sin 31°$

Practice A

In Exercises 1–3, find sin *F*, sin *G*, cos *F*, and cos *G*. Write each answer as a fraction and as a decimal rounded to four places.

1.



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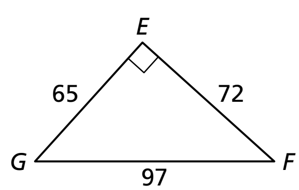
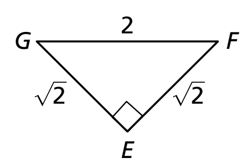


3.



In Exercises 4–6, write the expression in terms of cosine.

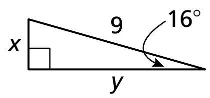
4. $\sin 9°$ **5.** $\sin 30°$ **6.** $\sin 77°$

In Exercises 7–9, write the expression in terms of sine.

7. $\cos 15^\circ$ **8.** $\cos 83^\circ$ **9.** $\cos 45^\circ$

In Exercises 10–13, find the value of each variable using sine and cosine. Round your answers to the nearest tenth.

10.



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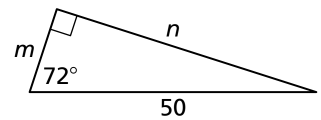

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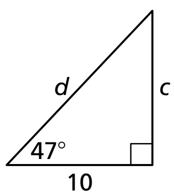

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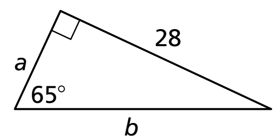


14. A camera attached to a kite is filming the damage caused by a brush fire in a closed-off area. The camera is directly above the center of the closed-off area.

a. A person is standing 100 feet away from the center of the closed-off area. The angle of depression from the camera to the person flying the kite is 25°. How long is the string on the kite?

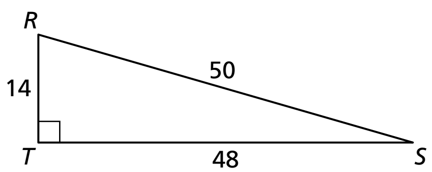
b. If the string on the kite is 200 feet long, how far away must the person flying the kite stand from the center of the closed-off area, assuming the same angle of depression of 25°, to film the damage?

Name ____ Date _________

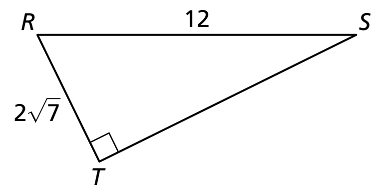
Practice B

In Exercises 1 and 2, find sin *R*, sin *S*, cos *R*, and cos *S*. Write each answer as a fraction and as a decimal rounded to four places.

1.



2.

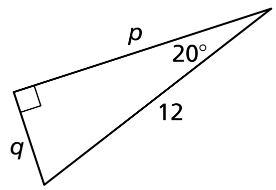


In Exercises 3–5, write the expression in terms of sine and/or cosine.

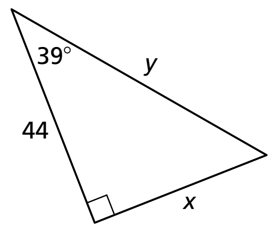
3. $\sin 7°$ **4.** $\cos 31°$ **5.** $\tan 60°$

In Exercises 6–8, find the value of each variable using sine and cosine. Round your answers to the nearest tenth.

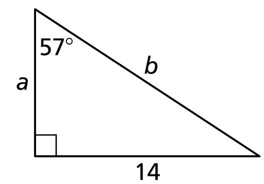
6.



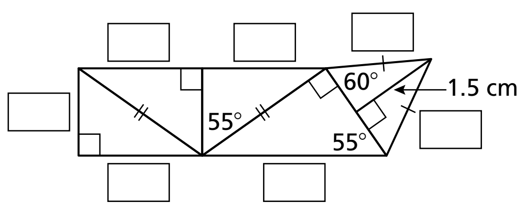
7.



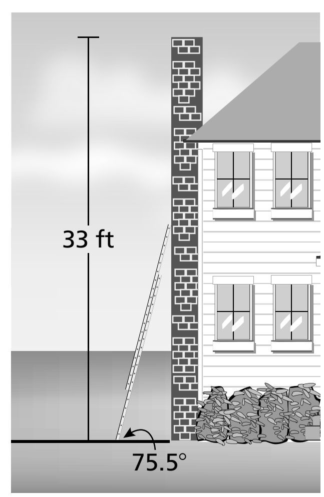
8.



9. Find the perimeter of the figure shown. Round your answer to the nearest centimeter.



10. You use an extension ladder to repair a chimney that is 33 feet tall. The length of the extension ladder ranges in one-foot increments from its minimum length to its maximum length. For safety reasons, you should always use an angle of about $75.5°$ between the ground and your ladder.



a. Your smallest extension ladder has maximum length of 17 feet. How high does this ladder safely reach on the chimney? Round your answer to the nearest tenth of a foot.

b. You place the ladder 3 feet from the base of the chimney. How many feet long should the ladder be? Round your answer to the nearest foot.

c. To reach the top of the chimney, you need a ladder that reaches 30 feet high. How many feet long should the ladder be? Round your answer to the nearest foot.