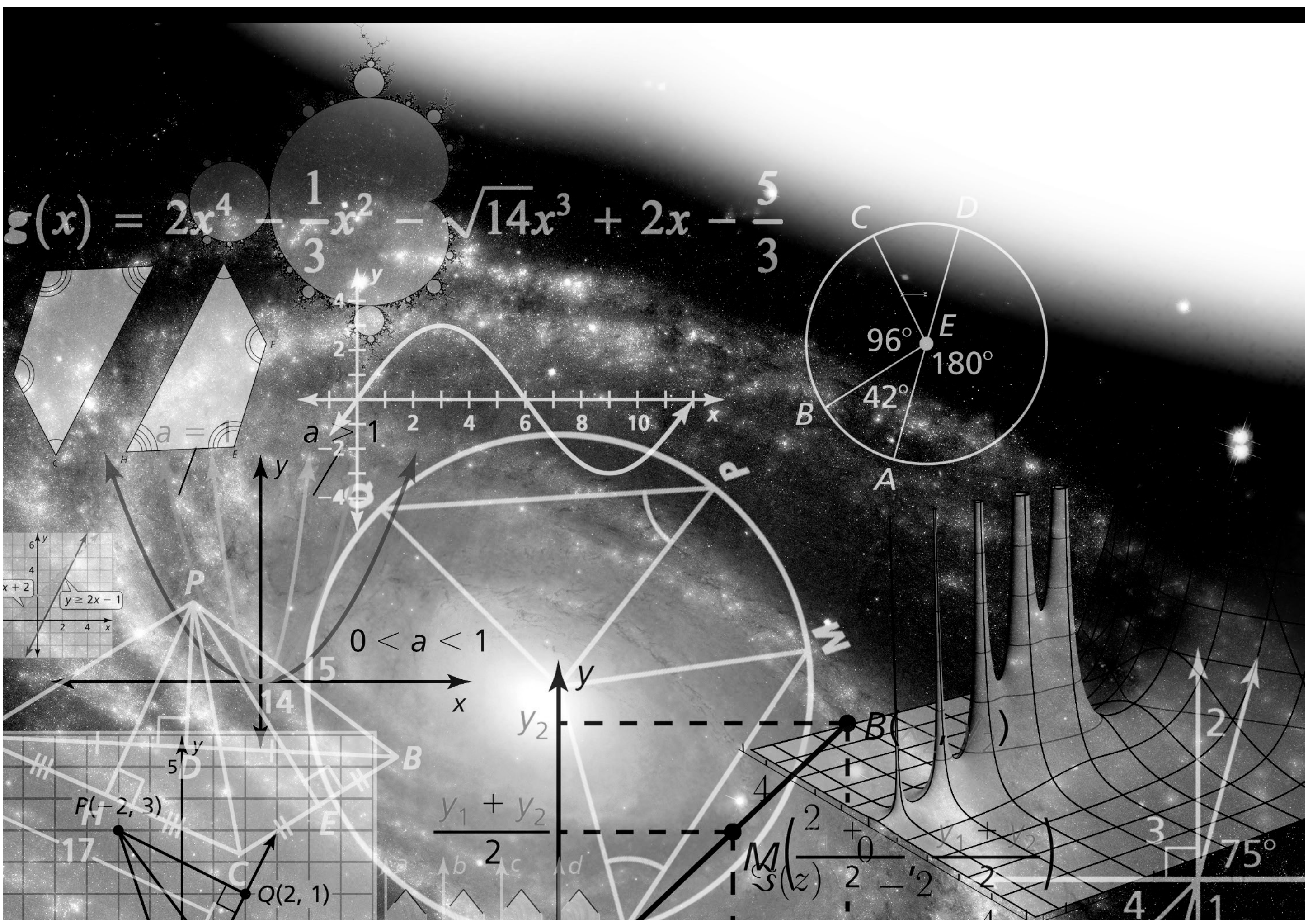
CHAPTER 11

Right Triangles and Trigonometry



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Chapter 11 Maintaining Mathematical Proficiency

Simplify the expression.

1. $\sqrt{500}$

2. $\sqrt{189}$

3. $\sqrt{252}$

4. $\dfrac{4}{\sqrt{3}}$

5. $\dfrac{11}{\sqrt{5}}$

6. $\dfrac{8}{\sqrt{2}}$

Solve the proportion.

7. $\dfrac{x}{21} = \dfrac{2}{7}$

8. $\dfrac{x}{5} = \dfrac{9}{4}$

9. $\dfrac{3}{x} = \dfrac{14}{42}$

10. $\dfrac{20}{27} = \dfrac{6}{x}$

11. $\dfrac{x - 4}{5} = \dfrac{10}{9}$

12. $\dfrac{15}{5x + 25} = \dfrac{3}{9}$

13. The Pythagorean Theorem states that $a^2 + b^2 = c^2$, where a and b are legs of a right triangle and c is the hypotenuse. Are you able to simplify the Pythagorean Theorem further to say that $a + b = c$? Explain.

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11.1 The Pythagorean Theorem

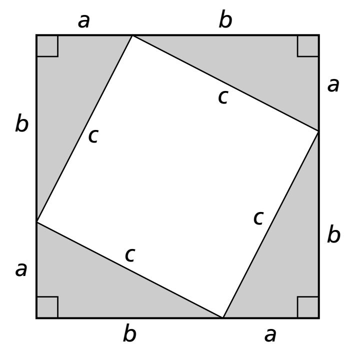
For use with Exploration 11.1

Essential Question How can you prove the Pythagorean Theorem?

1 EXPLORATION: Proving the Pythagorean Theorem without Words

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

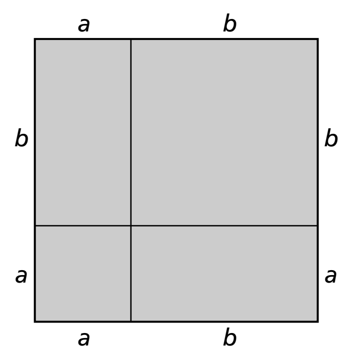
Work with a partner.



a. Draw and cut out a right triangle with legs a and b, and hypotenuse c.

b. Make three copies of your right triangle. Arrange all four triangles to form a large square as shown.

c. Find the area of the large square in terms of a, b, and c by summing the areas of the triangles and the small square.



d. Copy the large square. Divide it into two smaller squares and two equally-sized rectangles, as shown.

e. Find the area of the large square in terms of a and b by summing the areas of the rectangles and the smaller squares.

f. Compare your answers to parts (c) and (e). Explain how this proves the Pythagorean Theorem.

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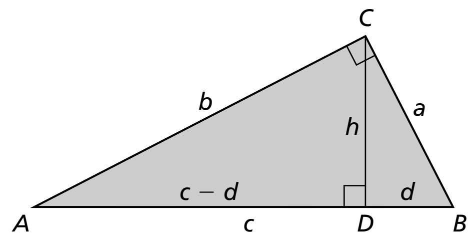
2 EXPLORATION: Proving the Pythagorean Theorem

Work with a partner.

a. Consider the triangle shown.



b. Explain why $\triangle ABC$, $\triangle ACD$, and $\triangle CBD$ are similar.

c. Write a two-column proof using the similar triangles in part (b) to prove that $a^2 + b^2 = c^2$.

Communicate Your Answer

3. How can you prove the Pythagorean Theorem?

4. Use the Internet or some other resource to find a way to prove the Pythagorean Theorem that is different from Explorations 1 and 2.

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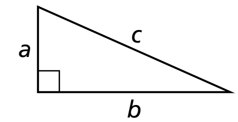
11.1 Practice
For use after Lesson 11.1

Theorems

Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$c^2 = a^2 + b^2$

Notes:

Core Concepts

Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
$3x$, $4x$, $5x$	$5x$, $12x$, $13x$	$8x$, $15x$, $17x$	$7x$, $24x$, $25x$

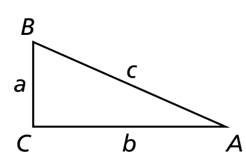
The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold-faced triple by the same factor.

Notes:

Theorems

Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

Notes:

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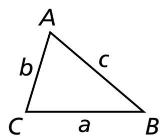
11.1 **Practice (continued)**

Pythagorean Inequalities Theorem

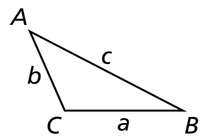
For any $\triangle ABC$, where c is the length of the longest side, the following statements are true.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



$c^2 < a^2 + b^2$

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



$c^2 > a^2 + b^2$

Notes:

Worked-Out Examples

Example #1

Find the value of x. Then tell whether the side lengths form a Pythagorean triple.

$c^2 = a^2 + b^2$

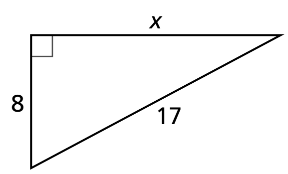
$17^2 = 8^2 + x^2$

$289 = 64 + x^2$

$225 = x^2$

$\sqrt{225} = x$

$15 = x$



Because the side lengths 8, 15, and 17 are integers that satisfy the equation $c^2 = a^2 + b^2$, they form a Pythagorean triple.

Example #2

Verify that the segment lengths form a triangle. Is the triangle acute, right, or obtuse?

6, 8, and 10

$6 + 8 \overset{?}{>} 10$ $\quad$ $6 + 10 \overset{?}{>} 8$ $\quad$ $8 + 10 \overset{?}{>} 6$

$14 > 10$ ✓ $\quad$ $16 > 8$ ✓ $\quad$ $18 > 6$ ✓

The segments with lengths 6, 8, and 10 form a triangle.

c^2 ___ $a^2 + b^2$

10^2 ___ $6^2 + 8^2$

100 ___ $36 + 64$

$100 = 100$

The triangle is a right triangle.

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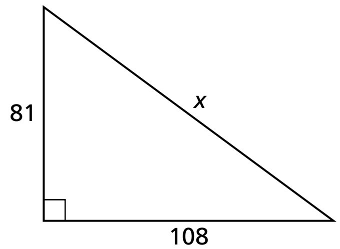
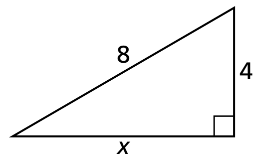
Practice A

In Exercises 1–6, find the value of *x*. Then tell whether the side lengths form a Pythagorean triple.

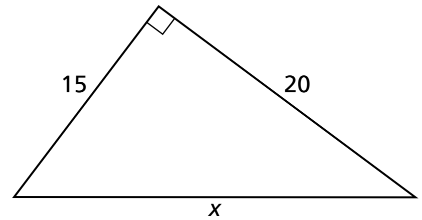
1.



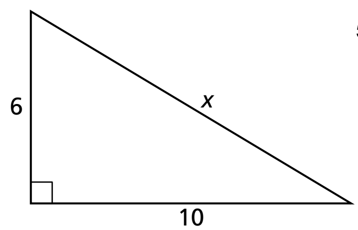
2.



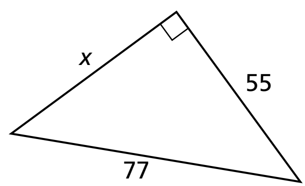
3.



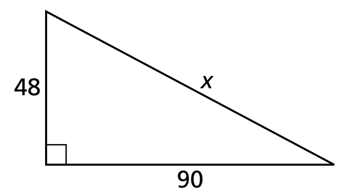
4.



5.



6.



7. From school, you biked 1.2 miles due south and then 0.5 mile due east to your house. If you had biked home on the street that runs directly diagonal from your school to your house, how many fewer miles would you have biked?

In Exercises 8 and 9, verify that the segment lengths form a triangle. Is the triangle *acute*, *right*, or *obtuse*?

8. 90, 216, and 234

9. 1, 1, and $\sqrt{3}$

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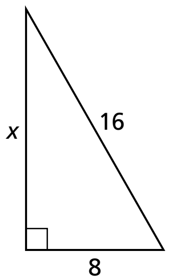
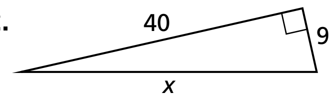
Practice B

In Exercises 1–3, find the value of *x*. Then tell whether the side lengths form a Pythagorean triple.

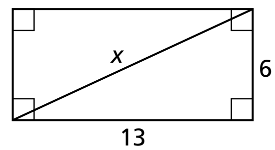
1.



2.

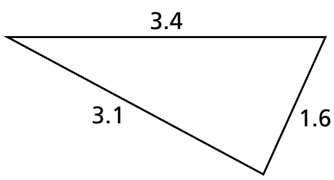


3.

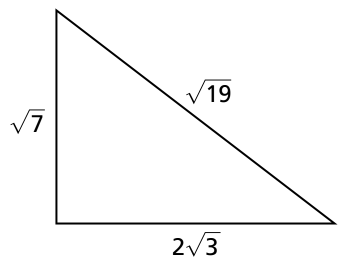


In Exercises 4 and 5, tell whether the triangle is a right triangle.

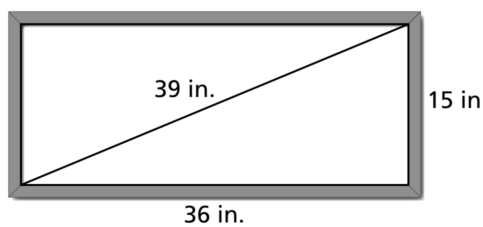
4.



5.



6. You construct a picture frame with a diagonal piece attached to the back for support, as shown. Can you tell from the dimensions whether the corners of the frame are right angles? Explain.



In Exercises 7–9, verify that the segment lengths form a triangle. Is the triangle *acute*, *right*, or *obtuse*?

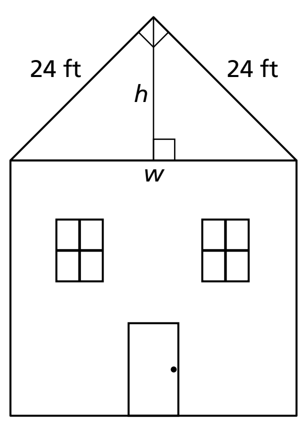
7. 14, 48, and 50

8. 7.1, 13.3, and 19.5

9. $\sqrt{67}$, 4, and 9

10. A triangle has side lengths of 12 feet and 18 feet. Your friend claims that the third side must be greater than 6 feet. Is your friend correct? Explain.

11. The diagram shows the design of a house roof. Each side of the roof is 24 feet long, as shown. Use the Pythagorean Theorem to answer each question.



a. What is the approximate width *w* of the house?

b. What is the approximate height *h* of the roof above the ceiling?

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