Name ____ Date __________

10.6 Proportionality Theorems

For use with Exploration 10.6

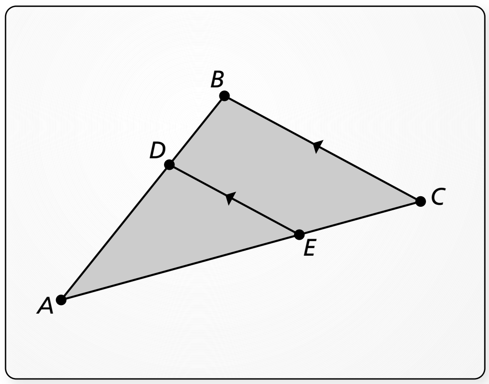
Essential Question What proportionality relationships exist in a triangle intersected by an angle bisector or by a line parallel to one of the sides?

1 EXPLORATION: Discovering a Proportionality Relationship

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$.

a. Construct $\overline{DE}$ parallel to $\overline{BC}$ with endpoints on $\overline{AB}$ and $\overline{AC}$, respectively.



b. Compare the ratios of AD to BD and AE to CE.

c. Move $\overline{DE}$ to other locations parallel to $\overline{BC}$ with endpoints on $\overline{AB}$ and $\overline{AC}$, and repeat part (b).

d. Change $\triangle ABC$ and repeat parts (a)–(c) several times. Write a conjecture that summarizes your results.

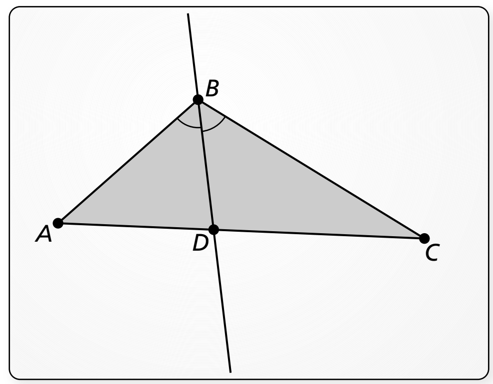
Name____ Date __________

2 EXPLORATION: Discovering a Proportionality Relationship

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$.

a. Bisect $\angle B$ and plot point D at the intersection of the angle bisector and $\overline{AC}$.



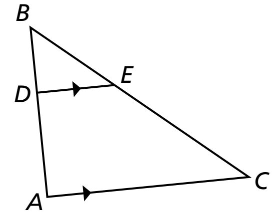
b. Compare the ratios of AD to DC and BA to BC.

c. Change $\triangle ABC$ and repeat parts (a) and (b) several times. Write a conjecture that summarizes your results.

Communicate Your Answer

3. What proportionality relationships exist in a triangle intersected by an angle bisector or by a line parallel to one of the sides?

4. Use the figure at the right to write a proportion.



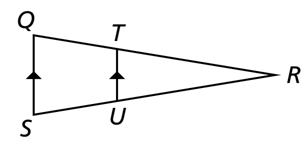
Name ____ Date __________

10.6 Practice
For use after Lesson 10.6

Theorems

Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

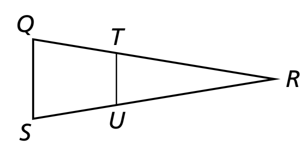


Notes:

If $\overline{TU} \parallel \overline{QS}$, then $\dfrac{RT}{TQ} = \dfrac{RU}{US}$.

Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

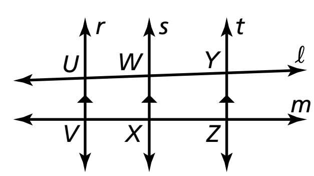


Notes:

If $\dfrac{RT}{TQ} = \dfrac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

Three Parallel Lines Theorem

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

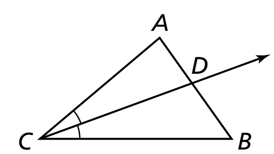


Notes:

$$\frac{UW}{WY} = \frac{VX}{XZ}$$

Triangle Angle Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



Notes:

$$\frac{AD}{DB} = \frac{CA}{CB}$$

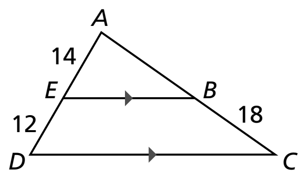
Name____ Date__________

10.6 **Practice (continued)**

Worked-Out Examples

Example #1

Find the length of $\overline{AB}$.



$\frac{AE}{ED} = \frac{AB}{BC}$

$\frac{14}{12} = \frac{AB}{18}$

$\frac{7}{6} = \frac{AB}{18}$

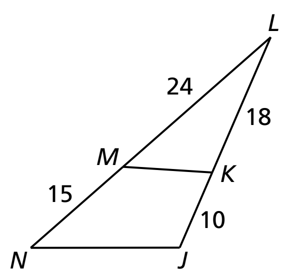
$\frac{7 \bullet 18}{6} = AB$

$21 = AB$

The length of $\overline{AB}$ is 21 units.

Example #2

Determine whether $\overline{KM} \parallel \overline{JN}$.



If $\frac{LM}{MN} = \frac{LK}{KJ}$, then $\overline{KM} \parallel \overline{JN}$.

$\frac{LM}{MN} = \frac{24}{15} = \frac{8}{5}$

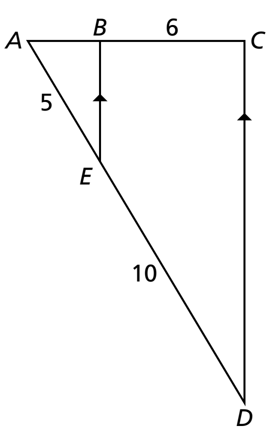
$\frac{LK}{KJ} = \frac{18}{10} = \frac{9}{5}$

Because $\frac{8}{5} \neq \frac{9}{5}$, $\overline{KM}$ is not parallel to $\overline{JN}$.

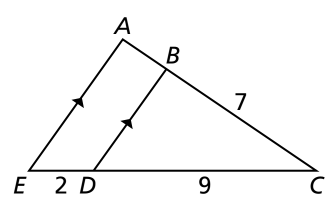
Practice A

In Exercises 1 and 2, find the length of $\overline{AB}$.

1.



2.

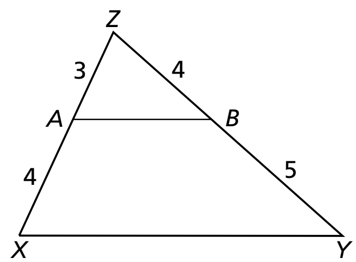


Name ______________________________ Date __________

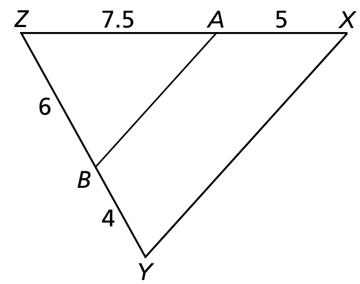
10.6 Practice (continued)

In Exercises 3 and 4, determine whether $\overline{AB} \parallel \overline{XY}$.

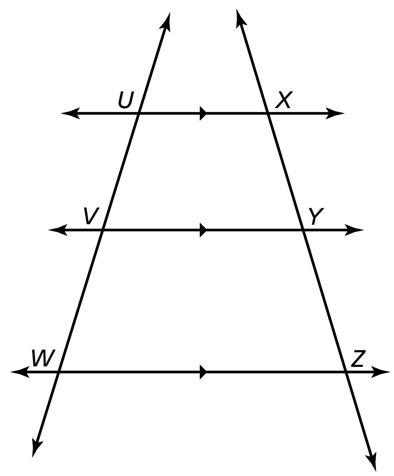
3.



4.



In Exercises 5–7, use the diagram to complete the proportion.



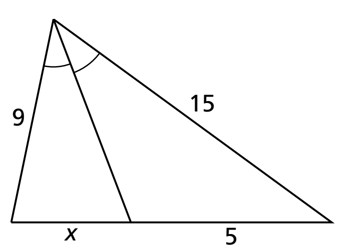
5. $\dfrac{UV}{UW} = \dfrac{XY}{\square}$

6. $\dfrac{XY}{YZ} = \dfrac{\square}{VW}$

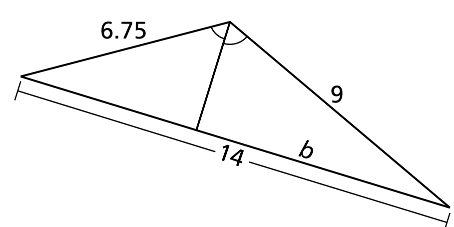
7. $\dfrac{\square}{ZY} = \dfrac{WU}{WV}$

In Exercises 8 and 9, find the value of the variable.

8.



9.

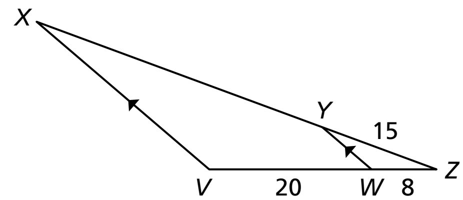


Name ____ Date _________

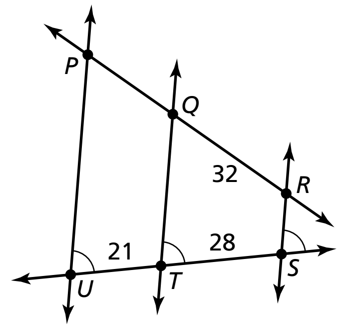
Practice B

In Exercises 1 and 2, find the length of the indicated line segment.

1. $\overline{XY}$

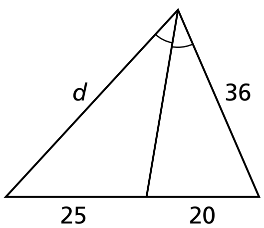


2. $\overline{PR}$

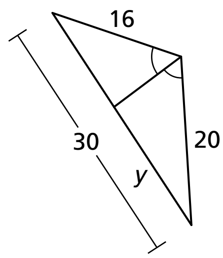


In Exercises 3 and 4, find the value of the variable.

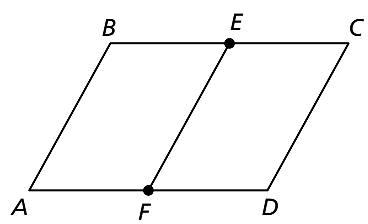
3.



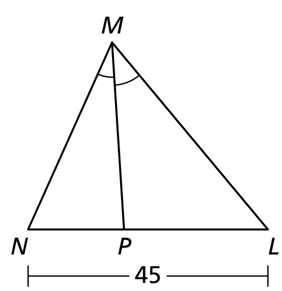
4.



5. The figure shows parallelogram $ABCD$, where E and F are the midpoints of $\overline{BC}$ and $\overline{AD}$ respectively. Your friend claims that $\overline{EF}$ is parallel to $\overline{AB}$ and $\overline{CD}$ by the Three Parallel Lines Theorem. Is your friend correct? Explain your reasoning.



6. The figure shows a triangle such that the length of $\overline{LP}$ is nine less than twice the length of $\overline{PN}$. Do you have enough information to find LP and PN? Explain your reasoning. If so, find LP and PN.



7. Use the diagram to write a two-column proof.

Given $\overline{WY}$ bisects $\angle XYZ$.
$\overline{YW}$ bisects $\angle XWZ$.
$YZ \cong WZ$

Prove $WXYZ$ is a kite.