

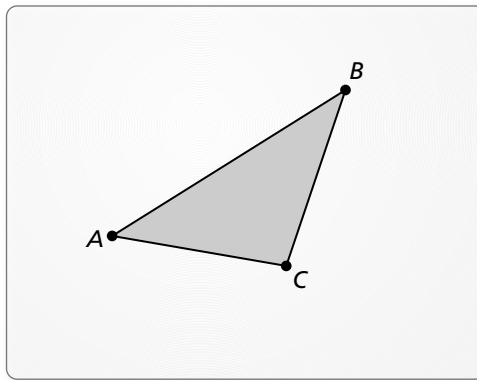
10.3**Similar Polygons**

For use with Exploration 10.3

Essential Question How are similar polygons related?**1 EXPLORATION:** Comparing Triangles after a Dilation

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$. Dilate $\triangle ABC$ to form a similar $\triangle A'B'C'$ using any scale factor k and any center of dilation.



- a. Compare the corresponding angles of $\triangle A'B'C'$ and $\triangle ABC$.

- b. Find the ratios of the lengths of the sides of $\triangle A'B'C'$ to the lengths of the corresponding sides of $\triangle ABC$. What do you observe?

- c. Repeat parts (a) and (b) for several other triangles, scale factors, and centers of dilation. Do you obtain similar results?

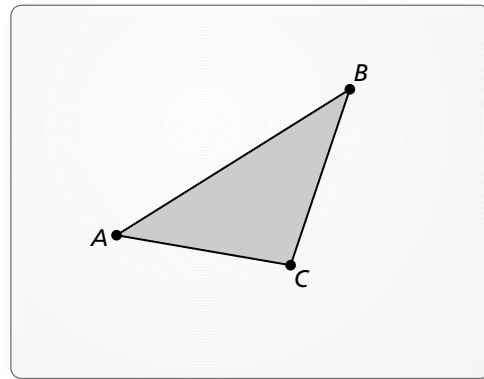
10.3 Similar Polygons (continued)**2 EXPLORATION: Comparing Triangles after a Dilation**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$. Dilate $\triangle ABC$ to form a similar $\triangle A'B'C'$ using any scale factor k and any center of dilation.

- a. Compare the perimeters of $\triangle A'B'C'$ and $\triangle ABC$. What do you observe?

- b. Compare the areas of $\triangle A'B'C'$ and $\triangle ABC$. What do you observe?



- c. Repeat parts (a) and (b) for several other triangles, scale factors, and centers of dilation. Do you obtain similar results?

Communicate Your Answer

3. How are similar polygons related?

4. A $\triangle RST$ is dilated by a scale factor of 3 to form $\triangle R'S'T'$. The area of $\triangle RST$ is 1 square inch. What is the area of $\triangle R'S'T'$?

10.3

Practice

For use after Lesson 10.3

Core Concepts

Corresponding Parts of Similar Polygons

In the diagram below, $\triangle ABC$ is similar to $\triangle DEF$. You can write “ $\triangle ABC$ is similar to $\triangle DEF$ ” as $\triangle ABC \sim \triangle DEF$. A similarity transformation preserves angle measure. So, corresponding angles are congruent. A similarity transformation also enlarges or reduces side lengths by a scale factor k . So, corresponding side lengths are proportional.



Corresponding angles

$$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$$

Ratios of corresponding side lengths

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$$

Notes:

Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

Notes:

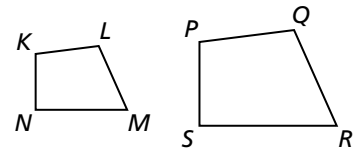
Theorems

Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

If $KLMN \sim PQRS$, then

$$\frac{PQ + QR + RS + SP}{KL + LM + MN + NK} = \frac{PQ}{KL} = \frac{QR}{LM} = \frac{RS}{MN} = \frac{SP}{NK}$$



Notes:

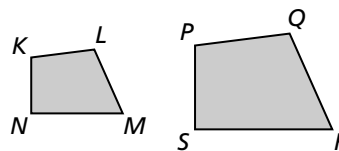
10.3 Practice (continued)

Areas of Similar Polygons

If two polygons are similar, then the ratio of their areas is equal to the squares of the ratios of their corresponding side lengths.

If $KLMN \sim PQRS$, then

$$\frac{\text{Area of } PQRS}{\text{Area of } KLMN} = \left(\frac{PQ}{KL}\right)^2 = \left(\frac{QR}{LM}\right)^2 = \left(\frac{RS}{MN}\right)^2 = \left(\frac{SP}{NK}\right)^2.$$



Notes:

Worked-Out Examples

Example #1

Find the scale factor. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality.

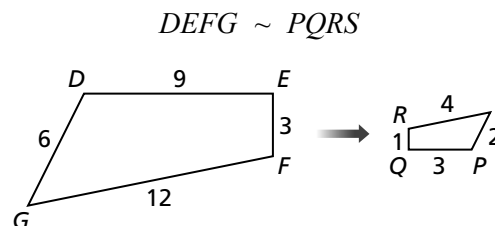
$$\frac{RS}{FG} = \frac{4}{12} = \frac{1}{3}$$

$$\frac{SP}{GD} = \frac{2}{6} = \frac{1}{3}$$

$$\frac{PQ}{DE} = \frac{3}{9} = \frac{1}{3}$$

$$\frac{QR}{EF} = \frac{1}{3}$$

The scale factor is $\frac{1}{3}$. The congruent angles are $\angle D \cong \angle P$, $\angle E \cong \angle Q$, $\angle F \cong \angle R$, and $\angle G \cong \angle S$. Because the ratios above are equal, $\frac{DE}{PQ} = \frac{EF}{QR} = \frac{FG}{RS} = \frac{GD}{SP}$.



Example #2

The polygons are similar. Find the value of x .

$$\frac{MN}{HJ} = \frac{NP}{JK}$$

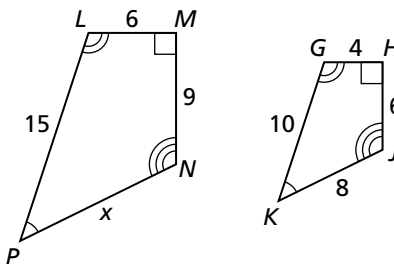
$$\frac{9}{6} = \frac{x}{8}$$

$$\frac{15}{10} = \frac{x}{8}$$

$$\frac{3}{2} = \frac{x}{8}$$

$$8 \cdot \frac{3}{2} = 8 \cdot \frac{x}{8}$$

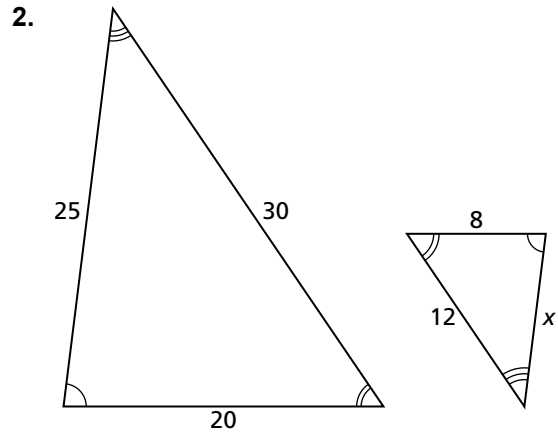
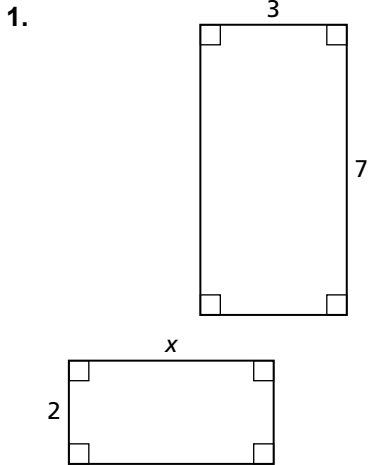
$$12 = x$$



10.3 Practice (continued)

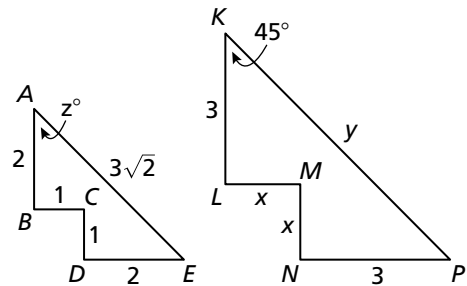
Practice A

In Exercises 1 and 2, the polygons are similar. Find the value of x .



In Exercises 3–8, $ABCDE \sim KLMNP$.

3. Find the scale factor from $ABCDE$ to $KLMNP$.
4. Find the scale factor from $KLMNP$ to $ABCDE$.
5. Find the values of x , y , and z .

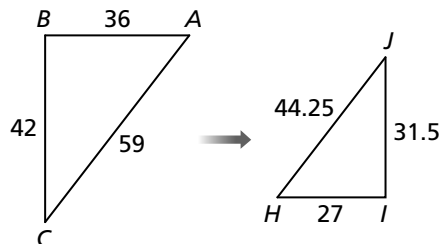


6. Find the perimeter of each polygon.
7. Find the ratio of the perimeters of $ABCDE$ to $KLMNP$.
8. Find the ratio of the areas of $ABCDE$ to $KLMNP$.

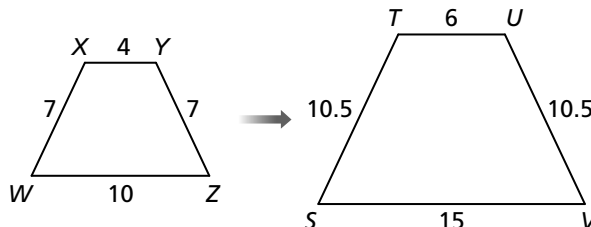
Practice B

In Exercises 1 and 2, find the scale factor. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality.

1. $\triangle ABC \sim \triangle HIJ$

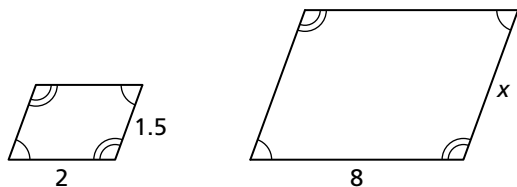


2. $WXYZ \sim STUV$

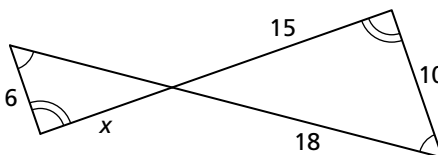


In Exercises 3 and 4, the polygons are similar. Find the value of x .

3.



4.



In Exercises 5 and 6, the figures are similar. Find the missing corresponding side length.

5. Figure A has a perimeter of 60 inches and one of the side lengths is 5 inches.
Figure B has a perimeter of 84 inches.
6. Figure A has an area of 4928 square feet and one of the side lengths is 88 feet.
Figure B has an area of 77 square feet.
7. In the diagram, $\triangle ABC \sim \triangle ADE$.

- a. Find the scale factor from $\triangle ABC$ to $\triangle ADE$.
- b. Find the value of x .
- c. Find $m\angle ABC$.
- d. The perimeter of $\triangle ABC$ is about 42.4 units.
Find the perimeter of the $\triangle ADE$.
- e. The area of $\triangle ABC$ is about 71.75 square units.
Find the area of the $\triangle ADE$.
- f. Is $\overline{BC} \parallel \overline{DE}$? Explain your reasoning.

