CHAPTER 10

Similarity

Name _____ Date _____

Maintaining Mathematical Proficiency

Tell whether the ratios form a proportion.

1. $\dfrac{3}{4}, \dfrac{16}{12}$

2. $\dfrac{35}{63}, \dfrac{45}{81}$
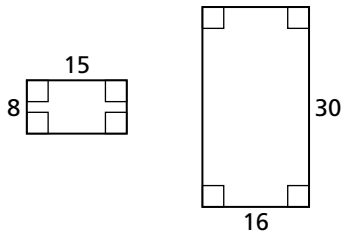
3. $\dfrac{12}{96}, \dfrac{16}{100}$

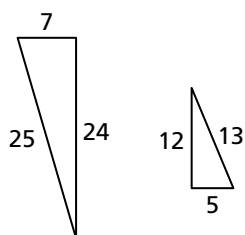4. $\dfrac{15}{24}, \dfrac{75}{100}$

5. $\dfrac{17}{68}, \dfrac{32}{128}$

6. $\dfrac{65}{105}, \dfrac{156}{252}$

**Tell whether the two figures are similar. Explain your reasoning.**

7.

15
8
30
16

8.

7
25 24
12 13
5

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10.1 Dilations
For use with Exploration 10.1

Essential Question What does it mean to dilate a figure?

┌───┐
1 **EXPLORATION:** Dilating a Triangle in a Coordinate Plane
└───┘

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

a. *Dilate* $\triangle ABC$ using a *scale factor* of 2 and a *center of dilation* at the origin to form $\triangle A'B'C'$. Compare the coordinates, side lengths, and angle measures of $\triangle ABC$ and $\triangle A'B'C'$.
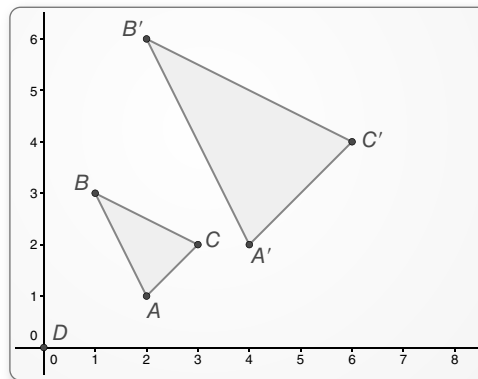
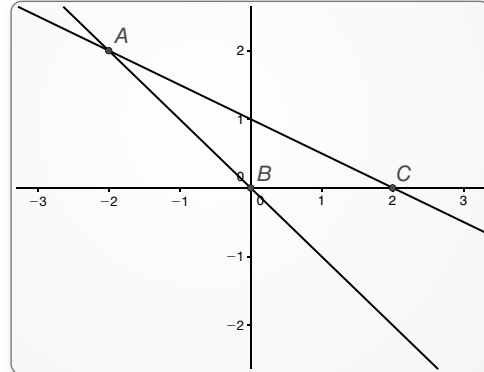**b.** Repeat part (a) using a *scale factor* of $\frac{1}{2}$.

c. What do the results of parts (a) and (b) suggest about the coordinates, side lengths, and angle measures of the image of $\triangle ABC$ after a dilation with a scale factor of k?

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2 **EXPLORATION:** Dilating Lines in a Coordinate Plane

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw $\overleftrightarrow{AB}$ that passes through the origin and $\overleftrightarrow{AC}$ that does not pass through the origin.

a. *Dilate* $\overleftrightarrow{AB}$ using a *scale factor* of 3 and a *center of dilation* at the origin. Describe the image.

b. *Dilate* $\overleftrightarrow{AC}$ using a *scale factor* of 3 and a *center of dilation* at the origin. Describe the image.

c. Repeat parts (a) and (b) using a scale factor of $\dfrac{1}{4}$.

d. What do you notice about dilations of lines passing through the center of dilation and dilations of lines not passing through the center of dilation?

Communicate Your Answer

3. What does it mean to dilate a figure?

4. Repeat Exploration 1 using a center of dilation at a point other than the origin.

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<table>
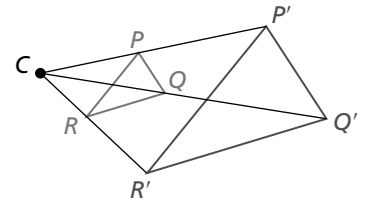<tr><td>**10.1**</td><td>**Practice**
For use after Lesson 10.1</td></tr>
</table>

Core Concepts

Dilations

A **dilation** is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the **center of dilation** and a **scale factor** k, which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation C and scale factor k maps every point P in a figure to a point P' so that the following are true.

- If P is the center point C, then $P = P'$.
- If P is not the center point C, then the image point P' lies on $\overrightarrow{CP}$.
  The scale factor k is a positive number such that $k = \dfrac{CP'}{CP}$.
- Angle measures are preserved.

Notes:

Coordinate Rule for Dilations

If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor k is the point $P'(kx, ky)$.
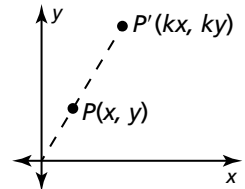
Notes:

Worked-Out Examples

Example #1

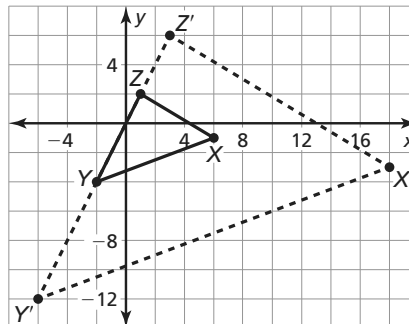**Graph the polygon and its image after a dilation with scale factor k.**

$X(6, -1), Y(-2, -4), Z(1, 2); \ k = 3$

$(x, y) \rightarrow (3x, 3y)$

$X(6, -1) \rightarrow X'(18, -3)$

$Y(-2, -4) \rightarrow Y'(-6, -12)$

$Z(1, 2) \rightarrow Z'(3, 6)$

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Example #2

Graph the polygon and its image after a dilation with scale factor k.
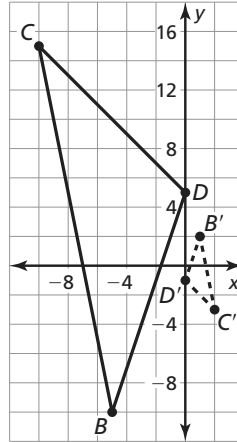
$B(-5, -10)$, $C(-10, 15)$, $D(0, 5)$; $k = -\dfrac{1}{5}$

$(x, y) \rightarrow \left(-\dfrac{1}{5}x, -\dfrac{1}{5}y\right)$
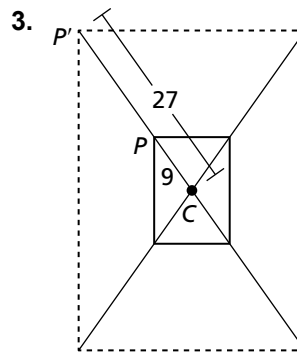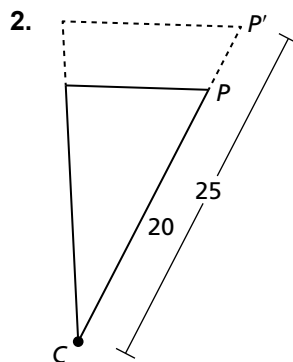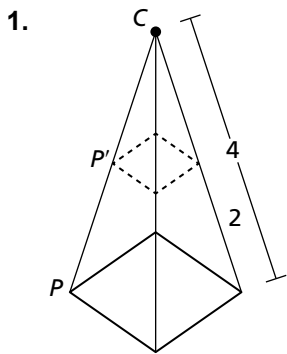
$B(-5, -10) \rightarrow B'(1, 2)$

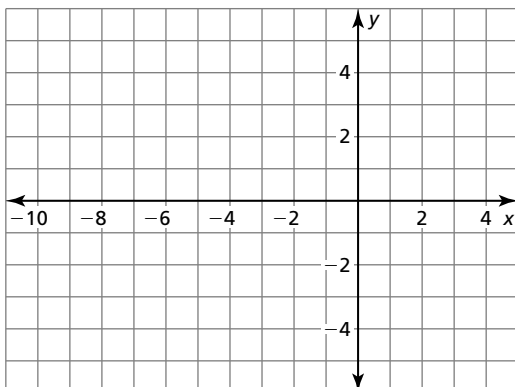$C(-10, 15) \rightarrow C'(2, -3)$

$D(0, 5) \rightarrow D'(0, -1)$

Practice A

In Exercises 1–3, find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.
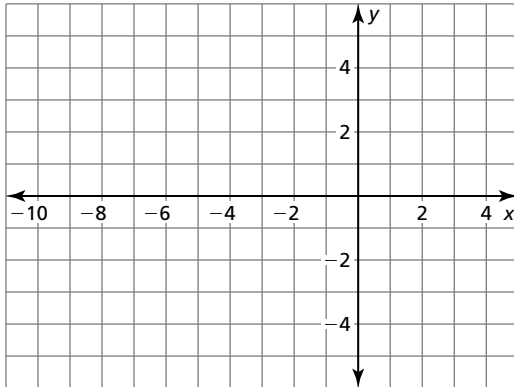
1.

2.

3.

In Exercises 4 and 5, graph the polygon and its image after a dilation with scale factor *k*.

4. $A(-3, 1)$, $B(-4, -1)$, $C(-2, -1)$; $k = 2$

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10.1 **Practice** (continued)

5. $P(-10, 0)$, $Q(-5, 0)$, $R(0, 5)$, $S(-5, 5)$; $k = \frac{1}{5}$

In Exercises 6 and 7, find the coordinates of the image of the polygon after a dilation with scale factor *k*.

**6.** $A(-3, 1)$, $B(-4, -1)$, $C(-2, -1)$; $k = -6$

7. $P(-8, 4)$, $Q(20, -8)$, $R(16, 4)$, $S(0, 12)$; $k = -0.25$

8. You design a poster on an 8.5-inch by 11-inch paper for a contest at your school. The poster of the winner will be printed on a 34-inch by 44-inch canvas to be displayed. What is the scale factor of this dilation?

9. A biology book shows the image of an insect that is 10 times its actual size. The image of the insect is 8 centimeters long. What is the actual length of the insect?

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Practice B

In Exercises 1 and 2, find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.

1.

2.
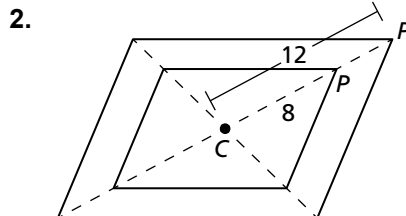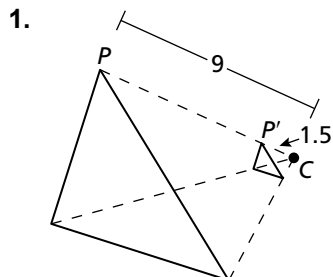
In Exercises 3 and 4, copy the diagram. Then use a compass and straightedge to construct a dilation with the given center and scale factor *k*.
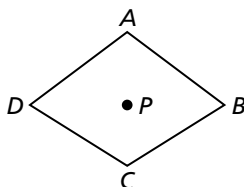
3. Center B, $k = 2$

4. Center P, $k = 75\%$

In Exercises 5 and 6, graph the polygon and its image after a dilation with a scale factor *k*.

5. $J(-3, 4)$, $K(2, 1)$, $L(3, -2)$, $M(-5, -4)$; $k = 50\%$

6. $V(1, 1)$, $W(-1, 0)$, $X(-4, 2)$, $Y(-3, 4)$, $Z(0, 3)$; $k = -3$

7. You look up at the sky at night and see the moon. It looks like it is about 2 millimeters across. If you then look at the moon through a telescope that has a magnification of 40 times, how big will it look to you through the telescope?

8. What would it mean for an object to be dilated with a scale factor of $k = 0$?

9. Your friend claims that if you dilate a rectangle by a certain scale factor, then the perimeter of the object also increases or decreases by the same factor. Is your friend correct? Explain your reasoning.

10. The image shows an object that has been dilated with an unknown scale factor. Use the given measures to determine the scale factor and solve for the value of x.