

9.5

Inequalities in Two Triangles

For use with Exploration 9.5

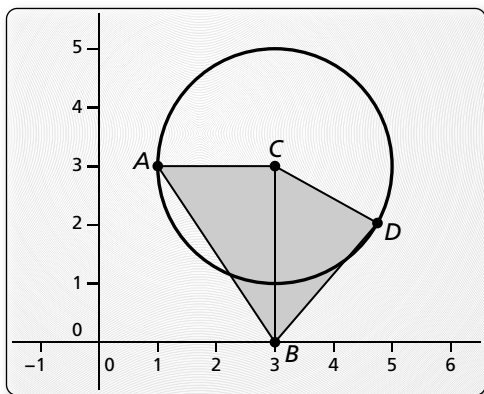
Essential Question If two sides of one triangle are congruent to two sides of another triangle, what can you say about the third sides of the triangles?

1 EXPLORATION: Comparing Measures in Triangles

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

- a. Draw $\triangle ABC$, as shown below.
- b. Draw the circle with center $C(3, 3)$ through the point $A(1, 3)$.
- c. Draw $\triangle DBC$ so that D is a point on the circle.



Sample

- Points
 $A(1, 3)$
 $B(3, 0)$
 $C(3, 3)$
 $D(4.75, 2.03)$
- Segments
 $BC = 3$
 $AC = 2$
 $DC = 2$
 $AB = 3.61$
 $DB = 2.68$

- d. Which two sides of $\triangle ABC$ are congruent to two sides of $\triangle DBC$? Justify your answer.
- e. Compare the lengths of \overline{AB} and \overline{DB} . Then compare the measures of $\angle ACB$ and $\angle DCB$. Are the results what you expected? Explain.
- f. Drag point D to several locations on the circle. At each location, repeat part (e). Copy and record your results in the table below.

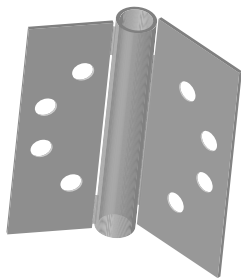
	D	AC	BC	AB	BD	$m\angle ACB$	$m\angle BCD$
1.	(4.75, 2.03)	2	3				
2.		2	3				
3.		2	3				
4.		2	3				
5.		2	3				

9.5 Inequalities in Two Triangles (continued)**1** **EXPLORATION:** Comparing Measures in Triangles (continued)

- g. Look for a pattern of the measures in your table. Then write a conjecture that summarizes your observations.

Communicate Your Answer

2. If two sides of one triangle are congruent to two sides of another triangle, what can you say about the third sides of the triangles?
3. Explain how you can use the hinge shown below to model the concept described in Question 2.



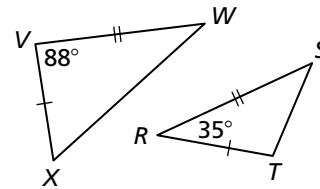
9.5

Practice
For use after Lesson 9.5

Theorems

Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.

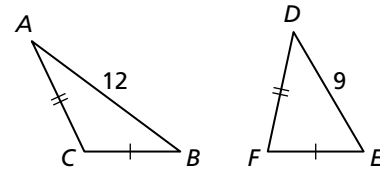


$WX > ST$

Notes:

Converse of the Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.



$m\angle C > m\angle F$

Notes:

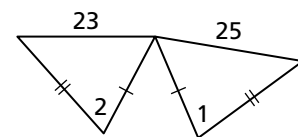
Worked-Out Examples

Example #1

Copy and complete the statement with $<$, $>$, or $=$. Explain your reasoning.

$m\angle 1 > \underline{\hspace{1cm}} m\angle 2$

$m\angle 1 > m\angle 2$; By the Converse of the Hinge Theorem, because $\angle 1$ is the included angle in the triangle with the longer third side, its measure is greater than that of $\angle 2$.



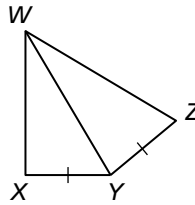
9.5 Practice (continued)

Example #2

Write a proof.

Given $\overline{XY} \cong \overline{YZ}$, $m\angle WYZ > m\angle WYX$

Prove $WZ > WX$

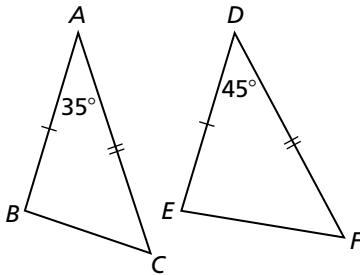


STATEMENTS	REASONS
1. $\overline{XY} \cong \overline{YZ}$	1. Given
2. $\overline{WY} \cong \overline{WY}$	2. Reflexive Property of Congruence
3. $m\angle WYZ > m\angle WYX$	3. Given
4. $WZ > WX$	4. Hinge Theorem

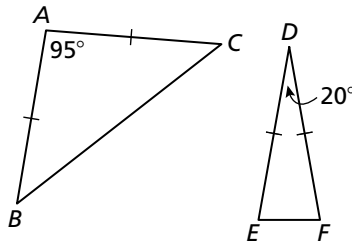
Practice A

In Exercises 1–9, complete the statement with $<$, $>$, or $=$. Explain your reasoning.

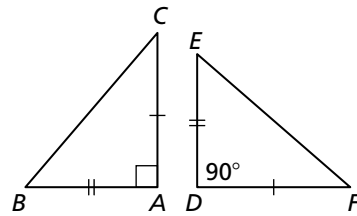
1. BC _____ EF



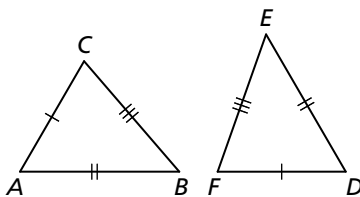
2. BC _____ EF



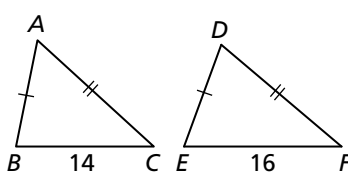
3. BC _____ EF



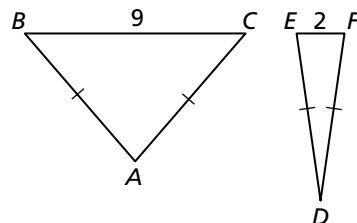
4. $m\angle A$ _____ $m\angle D$



5. $m\angle A$ _____ $m\angle D$

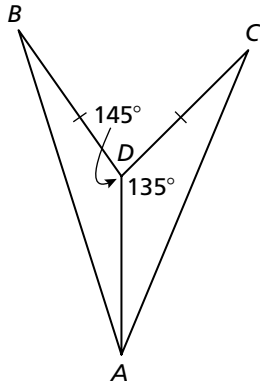


6. $m\angle A$ _____ $m\angle D$

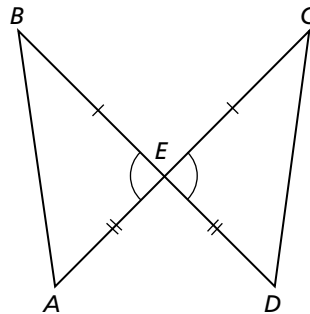


9.5 Practice (continued)

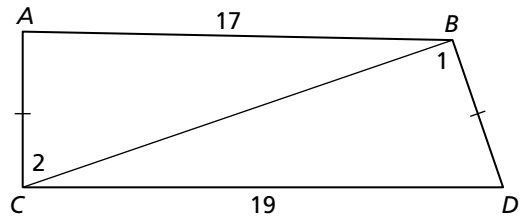
7. AB _____ AC



8. AB _____ CD



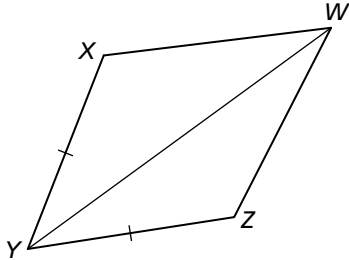
9. $m\angle 1$ _____ $m\angle 2$



In Exercises 10 and 11, write a proof.

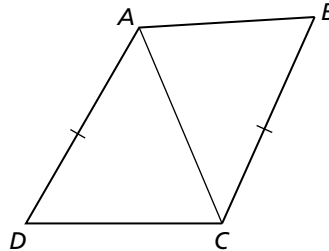
10. Given $\overline{XY} \cong \overline{YZ}$, $WX > WZ$

Prove $m\angle WYX > m\angle WYZ$



11. Given $\overline{AD} \cong \overline{BC}$, $m\angle DAC > m\angle ACB$

Prove $DC > AB$



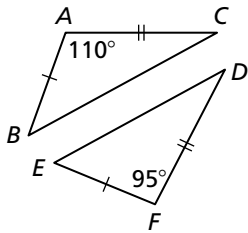
12. Loop a rubber band around the blade ends of a pair of scissors. Describe what happens to the rubber band as you open the scissors. How does that relate to the Hinge Theorem?

13. Starting from a point 10 miles north of Crow Valley, a crow flies northeast for 5 miles. Another crow, starting from a point 10 miles south of Crow Valley, flies due west for 5 miles. Which crow is farther from Crow Valley? Explain.

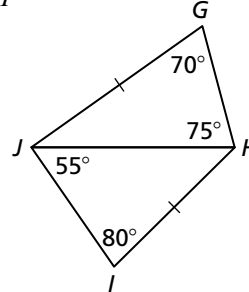
Practice B

In Exercises 1–4, copy and complete the statement with $<$, $>$, or $=$. Explain your reasoning.

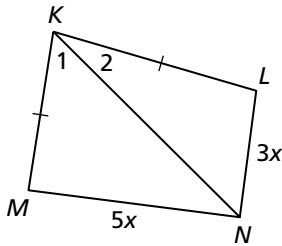
1. BC _____ DE



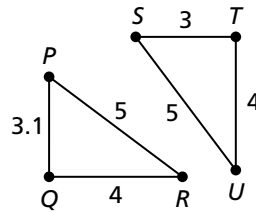
2. JI _____ GH



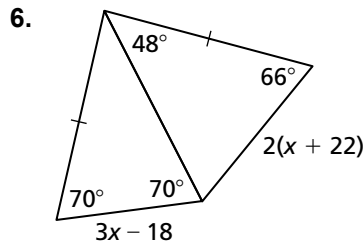
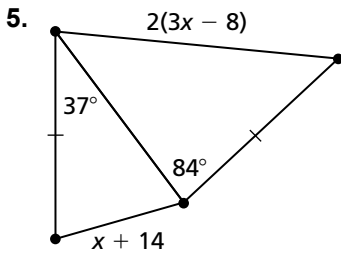
3. $m\angle 1$ _____ $m\angle 2$



4. $m\angle U$ _____ $m\angle R$



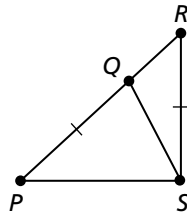
In Exercises 5 and 6, write and solve an inequality for the possible values of x .



7. Use the figure to write a proof.

Given $\overline{PQ} \cong \overline{SR}$

Prove $m\angle PQS > m\angle RSQ$



8. Two sailboats started at the same location. Sailboat A traveled 5 miles west, then turned 29° toward the north and continued for 8 miles. Sailboat B first went south for 8 miles, then turned 51° toward the east and continued for 5 miles. Which sailboat was farther from the starting point? Explain your reasoning.

9. How are the Hinge Theorem and the SAS Congruence Theorem similar? How are they different? Explain your reasoning.