Name_____ Date _____

# 9.3 The Triangle Midsegment Theorem

**For use with Exploration 9.3**

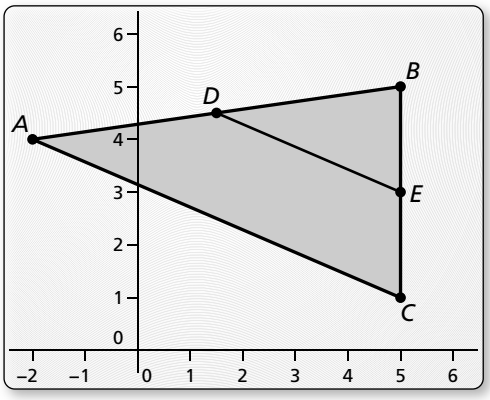
**Essential Question** How are the midsegments of a triangle related to the sides of the triangle?

## 1 EXPLORATION: Midsegments of a Triangle

**Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.**

**Work with a partner.** Use dynamic geometry software. Draw any $\triangle ABC$.

**a.** Plot midpoint $D$ of $\overline{AB}$ and midpoint $E$ of $\overline{BC}$. Draw $\overline{DE}$, which is a *midsegment* of $\triangle ABC$.




**Sample**
Points
$A(-2, 4)$
$B(5, 5)$
$C(5, 1)$
$D(1.5, 4.5)$
$E(5, 3)$
Segments
$BC = 4$
$AC = 7.62$
$AB = 7.07$
$DE = ?$

**b.** Compare the slope and length of $\overline{DE}$ with the slope and length of $\overline{AC}$.

**c.** Write a conjecture about the relationships between the midsegments and sides of a triangle. Test your conjecture by drawing the other midsegments of $\triangle ABC$, dragging vertices to change $\triangle ABC$, and noting whether the relationships hold.

Name _____ Date _____

## 9.3 The Triangle Midsegment Theorem (continued)

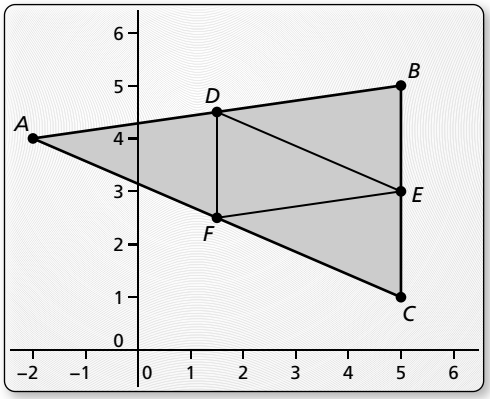
### 2 EXPLORATION: Midsegments of a Triangle

**Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.**

**Work with a partner.** Use dynamic geometry software. Draw any $\triangle ABC$.

**a.** Draw all three midsegments of $\triangle ABC$.

**b.** Use the drawing to write a conjecture about the triangle formed by the midsegments of the original triangle.




**Sample**

| Points | Segments |
|---|---|
| $A(-2, 4)$ | $BC = 4$ |
| $B(5, 5)$ | $AC = 7.62$ |
| $C(5, 1)$ | $AB = 7.07$ |
| $D(1.5, 4.5)$ | $DE = ?$ |
| $E(5, 3)$ | $DF = ?$ |
| | $EF = ?$ |

## Communicate Your Answer

**3.** How are the midsegments of a triangle related to the sides of the triangle?

**4.** In $\triangle RST$, $\overline{UV}$ is the midsegment connecting the midpoints of $\overline{RS}$ and $\overline{ST}$. Given $UV = 12$, find $RT$.

Name_____ Date_____

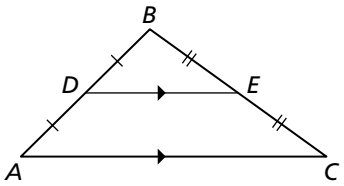
# 9.3 Practice
**For use after Lesson 9.3**

## Theorems

### Triangle Midsegment Theorem

The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.

$\overline{DE}$ is a midsegment of $\triangle ABC$, $\overline{DE} \parallel \overline{AC}$, and $DE = \frac{1}{2}AC$.

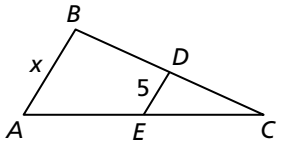



**Notes:**

## Worked-Out Examples

### Example #1

**$\overline{DE}$ is a midsegment of $\triangle ABC$. Find the value of x.**



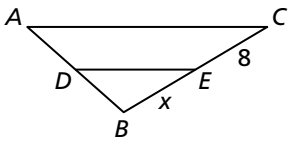

$DE = \frac{1}{2}AB$

$5 = \frac{1}{2}(AB)$

$x = 10$

### Example #2

**$\overline{DE}$ is a midsegment of $\triangle ABC$. Find the value of x.**




$BE = EC$

$x = 8$

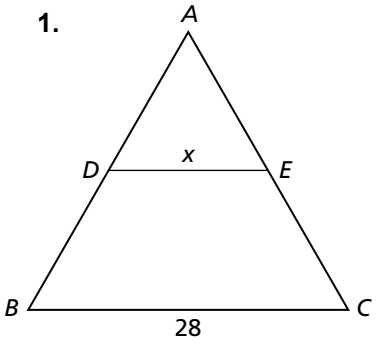
Name _____ Date _____

**9.3 Practice (continued)**

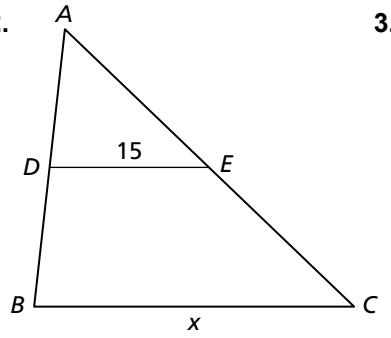
# Practice A

**In Exercises 1–3, *DE* is a midsegment of $\triangle ABC$. Find the value of *x*.**

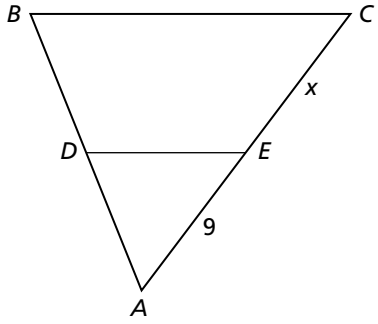
**1.**



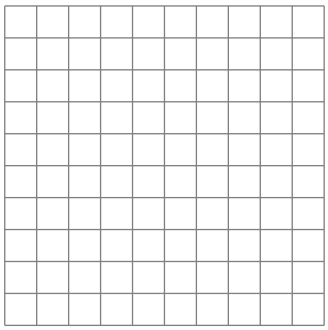

**2.**



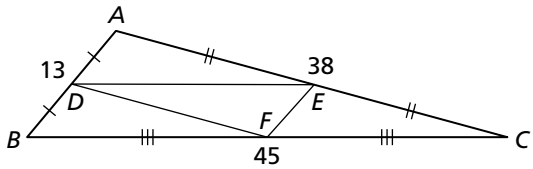

**3.**



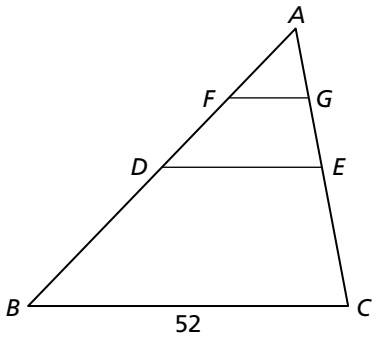

**4.** The vertices of a triangle are $A(-5, 6)$, $B(3, 8)$, and $C(1, -4)$. What are the vertices of the midsegment triangle?



**5.** What is the perimeter of $\triangle DEF$?




**6.** In the diagram, $\overline{DE}$ is a midsegment of $\triangle ABC$, and $\overline{FG}$ is a midsegment of $\triangle ADE$. Find $FG$.

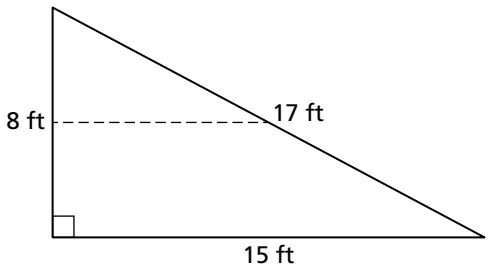


Name_____ Date _____

## 9.3 Practice (continued)

**7.** The area of $\triangle ABC$ is 48 $cm^2$. $\overline{DE}$ is a midsegment of $\triangle ABC$. What is the area of $\triangle ADE$?

**8.** The diagram below shows a triangular wood shed. You want to install a shelf halfway up the 8-foot wall that will be built between the two walls.



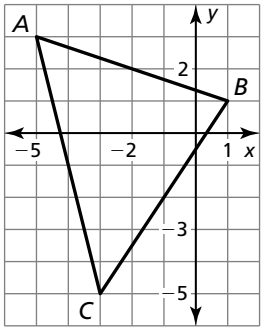

**a.** How long will the shelf be?

**b.** How many feet should you measure from the ground along the slanting wall to find where to attach the opposite end of the shelf so that it will be level?

Name_____ Date _____

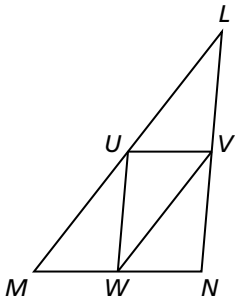
# Practice B

**In Exercises 1–4, use the graph of $\triangle ABC$.**

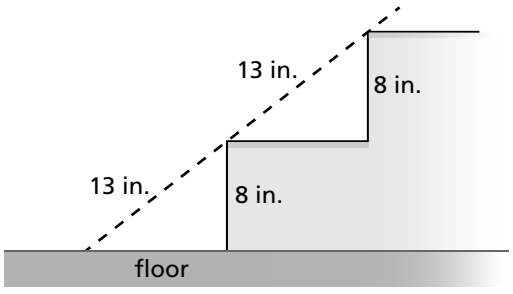



1. Find the coordinates of the midpoint $D$ of $\overline{AB}$, the midpoint $E$ of $\overline{CB}$, and the midpoint $F$ of $\overline{AC}$.
2. Graph the midsegment triangle, $\triangle DEF$.
3. Show that $\overline{FD} \parallel \overline{CB}$, $\overline{FE} \parallel \overline{AB}$, and $\overline{DE} \parallel \overline{AC}$.
4. Show that $FD = \frac{1}{2}CB$, $FE = \frac{1}{2}AB$, and $DE = \frac{1}{2}AC$.

**In Exercises 5–8, use $\triangle LMN$, where $U$, $V$, and $W$ are the midpoints of the sides.**




5. When $LV = 9$, what is $UW$?
6. When $LU = 2(x - 5)$ and $VW = 8 - x$, what is $LM$?
7. When $NL = 2x(12 + x)$ and $UW = (x + 4)^2$, what is $LV$?
8. When $UV = 2y + 14$ and $MN = 13 - y$, what is $WN$?
9. The bottom two steps of a stairwell are shown. Explain how to use the given measures to verify that the bottom step is parallel to the floor.




10. Your friend claims that a triangle with side lengths of $a$, $b$, and $c$ will have half the area of a triangle with side lengths of $2a$, $2b$, and $2c$. Is your friend correct? Explain your reasoning.