CHAPTER 9

Relationships Within Triangles

Name_____ Date _____

Write an equation of the line passing through point *P* that is perpendicular to the given line.

1. $P(5, 2), y = 2x + 6$
2. $P(4, 2), y = 6x - 3$
3. $P(-1, -2), y = -3x + 6$

4. $P(-8, 3), y = 3x - 1$
5. $P(6, 7), y = x - 5$
6. $P(3, 7), y = \frac{1}{4}x + 4$

Write the sentence as an inequality.

7. A number g is at least 4 and no more than 12.

8. A number r is more than 2 and less than 7.

9. A number q is less than or equal to 6 or greater than 1.

10. A number p is fewer than 17 or no less than 5.

11. A number k is greater than or equal to -4 and less than 1.

Name_____ Date _____

9.1 Proving Geometric Relationships
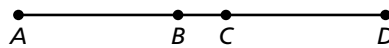
For use with Exploration 9.1

Essential Question How can you prove a mathematical statement?

A **proof** is a logical argument that uses deductive reasoning to show that a statement is true.

1 EXPLORATION: Writing Reasons in a Proof

Work with a partner. Four steps of a proof are shown. Write the reasons for each statement.

Given $AD = AB + AC$

Prove $CD = AB$

STATEMENTS	REASONS
1. $AD = AB + AC$	**1.** Given
2. $AC + CD = AD$	**2.** _____
3. $AC + CD = AB + AC$	**3.** _____
4. $CD = AB$	**4.** _____

2 EXPLORATION: Writing Steps in a Proof

Work with a partner. Five steps of a proof are shown. Complete the statements that correspond to each reason.

Given $m\angle ABD = m\angle CBE$

Prove $m\angle 1 = m\angle 3$

Name _____ Date _____

2 **EXPLORATION:** Writing Steps in a Proof (continued)

STATEMENTS	REASONS
1. $m\angle ABD = m\angle 1 + m\angle 2$	1. Angle Addition Postulate
2. $m\angle CBE = $ _____	2. Angle Addition Postulate
3. _____	3. Given
4. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	4. Substitution Property of Equality
5. _____	5. Subtraction Property of Equality

Communicate Your Answer

3. How can you prove a mathematical statement?

4. In Exploration 2, can you prove that $m\angle 1 = m\angle 2$? Explain your reasoning.

Name_____ Date_____

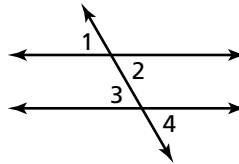**Notes:**

Worked-Out Examples

Example #1

Copy and complete the proof.

Statement about congruent angles

Given $\angle 1 \cong \angle 3$

Prove $\angle 2 \cong \angle 4$

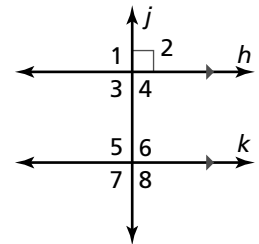| STATEMENTS	REASONS
1. $\angle 1 \cong \angle 3$	**1.** Given
2. $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$	**2.** Vertical Angles Congruence Theorem
3. $\angle 2 \cong \angle 3$ (or $\angle 4 \cong \angle 1$)	**3.** Transitive Property of Congruence
4. $\angle 2 \cong \angle 4$	**4.** Transitive Property of Congruence

Example #2

Copy and complete the paragraph proof. Then write a two-column proof.

Perpendicular Transversal Theorem

Given $h \parallel k, j \perp h$

Prove $j \perp k$

Line h and line k are parallel, and line j and line h are perpendicular. By the definition of perpendicular lines, $m\angle 2 = \underline{\quad}$. By the _____ , $\angle 2 \cong \angle 6$. By the definition of congruent angles, _____ . By the Transitive Property of Equality, _____ = 90°. By the _____ ____, $j \perp k$.

Paragraph Proof

Given $h \parallel k, j \perp h$

Prove $j \perp k$

Line h and line k are parallel, and line j and line h are perpendicular. By the definition of perpendicular lines, $m\angle 2 = 90°$. By the Corresponding Angles Theorem, $\angle 2 \cong \angle 6$. By the definition of congruent angles, $m\angle 2 = m\angle 6$. By the Transitive Property of Equality, $m\angle 6 = 90°$. By the definition of perpendicular lines, $j \perp k$.

9.1   **Practice** (continued)

Two-Column Proof

STATEMENTS	REASONS
1. $h \parallel k, j \perp h$	1. Given
2. $m\angle 2 = 90°$	2. Definition of perpendicular lines
3. $\angle 2 \cong \angle 6$	3. Corresponding Angles Theorem
4. $m\angle 2 = m\angle 6$	4. Definition of congruent angles
5. $m\angle 6 = 90°$	5. Transitive Property of Equality
6. $j \perp k$	6. Definition of perpendicular lines

Practice A

1. Complete the proof.

   Given $\angle AEB$ is a complement of $\angle BEC$.

   Prove $m\angle AED = 90°$
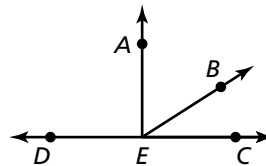
STATEMENTS	REASONS
1. $\angle AEB$ is a complement of $\angle BEC$.	1. Given
2. _____	2. Definition of complementary angles
3. $m\angle AEC = m\angle AEB + m\angle BEC$	3. _____
4. $m\angle AEC = 90°$	4. _____
5. $m\angle AED + m\angle AEC = 180°$	5. Definition of supplementary angles
6. _____	6. Substitution Property of Equality
7. $m\angle AED = 90°$	7. _____

Name_____ Date_____

2. Write a two-column proof.

Given M is the midpoint of $\overline{RT}$.

Prove $MT = RS + SM$

STATEMENTS	REASONS

Name_____ Date _____
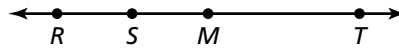
3. Complete the flowchart proof. Then write a two-column proof.

   Given ∠*ACB* and ∠*ACD* are supplementary.
   ∠*EGF* and ∠*ACD* are supplementary.

   Prove ∠*ACB* ≅ ∠*EGF*

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┌─────────────────────┐      ┌───────────────────────────────┐      ┌───────────────────────────┐
│ ∠ACB and ∠ACD       │ ───▶ │ m∠ACB + m∠ACD = 180°          │ ───▶ │                           │
│ are supplementary.  │      └───────────────────────────────┘      └───────────────────────────┘
└─────────────────────┘                                                          │  Transitive Property
        Given            ┌───────────────────────┐                               │  of Equality
                         │                       │                               ▼
                         └───────────────────────┘      ┌───────────────────────────┐
                                                         │ m∠ACB = m∠EGF             │
┌─────────────────────┐      ┌───────────────────────────────┐      └───────────────────────────┘
│                     │ ───▶ │ m∠EGF + m∠ACD = 180°          │               │  Subtraction Property
└─────────────────────┘      └───────────────────────────────┘               │  of Equality
        Given            ┌───────────────────────┐                           ▼
                         │                       │              ┌───────────────────────────┐
                         └───────────────────────┘              │ ∠ACB ≅ ∠EGF               │
                                                                └───────────────────────────┘
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Two-Column Proof

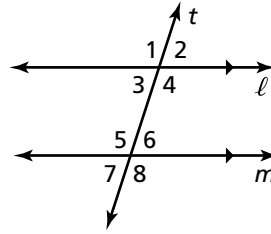| STATEMENTS	REASONS

Name_____ Date_____
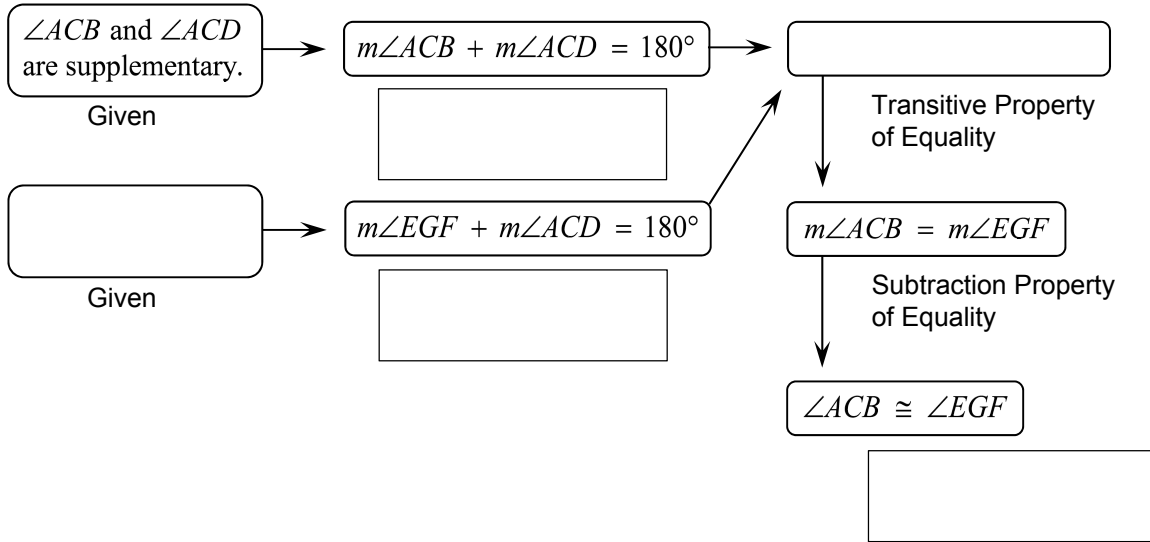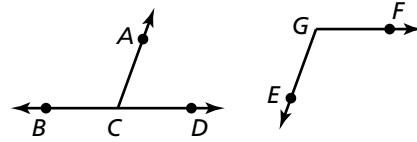
Practice B

1. Copy and complete the proof of the
Alternate Interior Angles Theorem.

Given $\ell \parallel m$
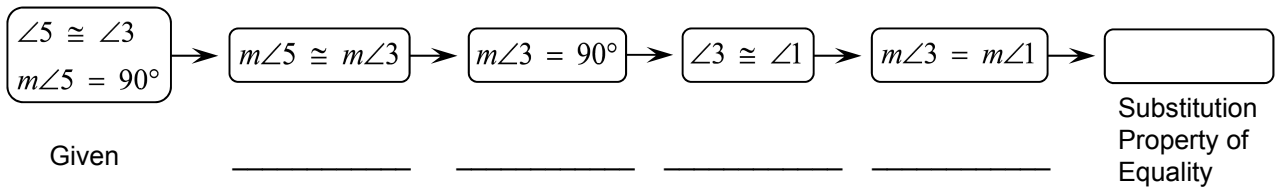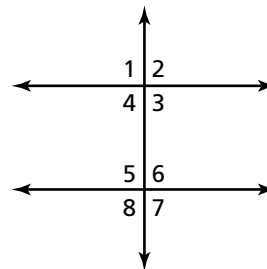
Prove $\angle 4 \cong \angle 5$

STATEMENTS	REASONS
1. $\ell \parallel m$	1. Given
2. $\angle 4 \cong \angle 8$	2. _____
3. _____	3. Vertical Angles Congruence Theorem
4. $\angle 4 \cong \angle 5$	4. Transitive Property of Congruence

2. Copy and complete the flowchart proof.
Then write a paragraph proof.

Given $\angle 5 \cong \angle 3$

$\qquad m\angle 5 = 90°$

Prove $m\angle 1 = 90°$

$\boxed{\begin{array}{c}\angle 5 \cong \angle 3 \\ m\angle 5 = 90°\end{array}} \rightarrow \boxed{m\angle 5 \cong m\angle 3} \rightarrow \boxed{m\angle 3 = 90°} \rightarrow \boxed{\angle 3 \cong \angle 1} \rightarrow \boxed{m\angle 3 = m\angle 1} \rightarrow \boxed{}$

Given _____ _____ _____ _____

Substitution
Property of
Equality

3. Your friend says that there is
not enough information to prove
that $\angle ABD \cong \angle CBD$. Is your
friend correct? Explain your
reasoning.