Name ____ Date __________

7.3 Rotations

For use with Exploration 7.3

Essential Question How can you rotate a figure in a coordinate plane?

1 EXPLORATION: Rotating a Triangle in a Coordinate Plane

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

b. *Rotate* the triangle 90° counterclockwise about the origin to form $\triangle A'B'C'$.

c. What is the relationship between the coordinates of the vertices of $\triangle ABC$ and those of $\triangle A'B'C'$?

d. What do you observe about the side lengths and angle measures of the two triangles?

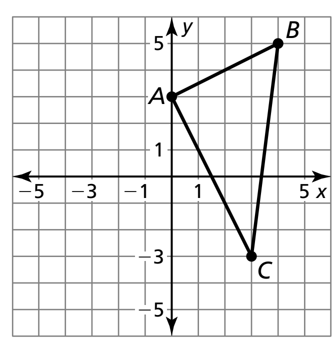
2 EXPLORATION: Rotating a Triangle in a Coordinate Plane

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. The point (x, y) is rotated 90° counterclockwise about the origin. Write a rule to determine the coordinates of the image of (x, y).

b. Use the rule you wrote in part (a) to rotate $\triangle ABC$ 90° counterclockwise about the origin. What are the coordinates of the vertices of the image, $\triangle A'B'C'$?



c. Draw $\triangle A'B'C'$. Are its side lengths the same as those of $\triangle ABC$? Justify your answer.

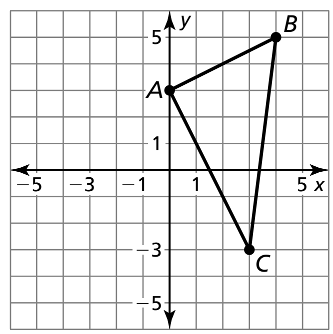
Name____ Date __________

3 EXPLORATION: Rotating a Triangle in a Coordinate Plane

Work with a partner.

a. The point (x, y) is rotated 180° counterclockwise about the origin. Write a rule to determine the coordinates of the image of (x, y). Explain how you found the rule.

b. Use the rule you wrote in part (a) to rotate $\triangle ABC$ (from Exploration 2) 180° counterclockwise about the origin. What are the coordinates of the vertices of the image, $\triangle A'B'C'$?



Communicate Your Answer

4. How can you rotate a figure in a coordinate plane?

5. In Exploration 3, rotate $\triangle A'B'C'$ 180° counterclockwise about the origin. What are the coordinates of the vertices of the image, $\triangle A''B''C''$? How are these coordinates related to the coordinates of the vertices of the original triangle, $\triangle ABC$?

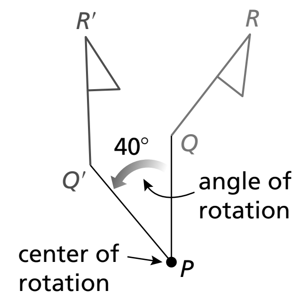
Name ______________________________ Date __________

7.3 Practice

For use after Lesson 7.3

Core Concepts

Rotations



A **rotation** is a transformation in which a figure is turned about a fixed point called the **center of rotation**. Rays drawn from the center of rotation to a point and its image form the **angle of rotation**.

A rotation about a point P through an angle of $x°$ maps every point Q in the plane to a point Q', so that one of the following properties is true.

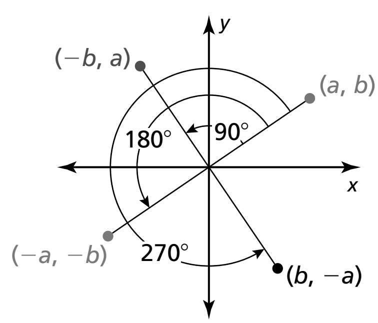
- If Q is not the center of rotation P, then $QP = Q'P$ and $m\angle QPQ' = x°$, or
- If Q is the center of rotation P, then $Q = Q'$.

Notes:

Coordinate Rules for Rotations about the Origin

When a point (a, b) is rotated counterclockwise about the origin, the following are true.

- For a rotation of $90°$, $(a, b) \rightarrow (-b, a)$.
- For a rotation of $180°$, $(a, b) \rightarrow (-a, -b)$.
- For a rotation of $270°$, $(a, b) \rightarrow (b, -a)$.



Notes:

Rotation Postulate

A rotation is a rigid motion.

Name______________________________ Date__________

7.3 **Practice (continued)**

Worked-Out Examples

Example #1

Graph the polygon and its image after a rotation of the given number of degrees about the origin.

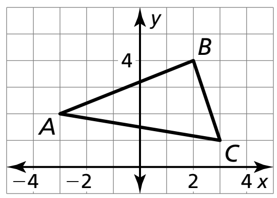
Use the coordinate rule for a 90° rotation around the origin, $(a, b) \rightarrow (-b, a)$.

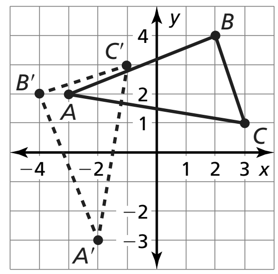
$A(-3, 2) \rightarrow A'(-2, -3)$

$B(2, 4) \rightarrow B'(-4, 2)$

$C(3, 1) \rightarrow C'(-1, 3)$

90°





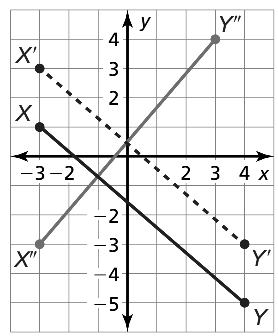
Example #2

Graph $\overline{XY}$ with endpoints X(−3, 1) and Y(4, −5) and its image after the composition.

Translation: $(x, y) \rightarrow (x, y+2)$

Rotation: 90° about the origin

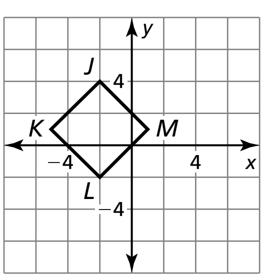
The translated image has endpoints $X'(-3, 3)$ and $Y'(4, -3)$. After the rotation of 90° about the origin, the endpoints are $X''(-3, -3)$ and $Y''(3, 4)$.



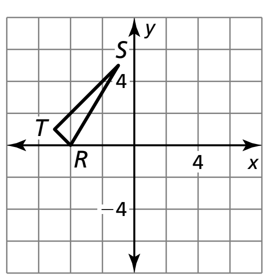
Practice A

In Exercises 1–3, graph the image of the polygon after a rotation of the given number of degrees about the origin.

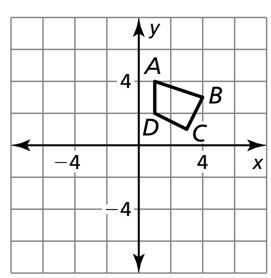
1. 180°



2. 90°



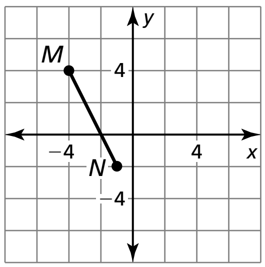
3. 270°



In Exercises 4–7, graph the image of $\overline{MN}$ after the composition.

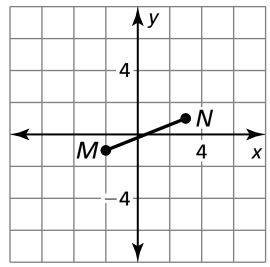
4. **Reflection:** x-axis

Rotation: 180° about the origin



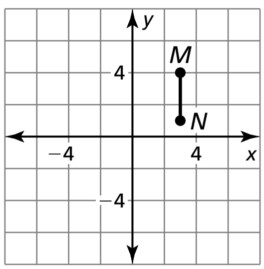
5. **Rotation:** 90° about the origin

Translation: $(x, y) \rightarrow (x + 2, y - 3)$



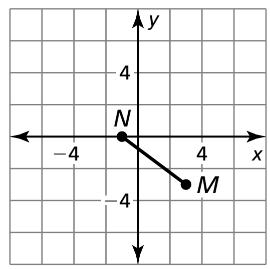
6. **Rotation:** 270° about the origin

Reflection: in the line $y = x$



7. **Rotation:** 90° about the origin

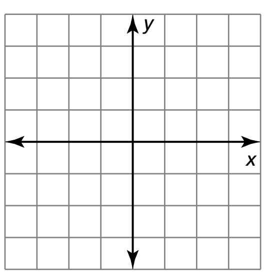
Translation: $(x, y) \rightarrow (x - 5, y)$



In Exercises 8 and 9, graph $\triangle JKL$ with vertices $J(2, 3)$, $K(1, -1)$, and $L(-1, 0)$ and its image after the composition.

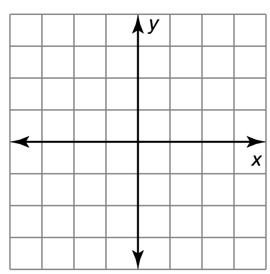
8. **Rotation:** 180° about the origin

Reflection: $x = 2$



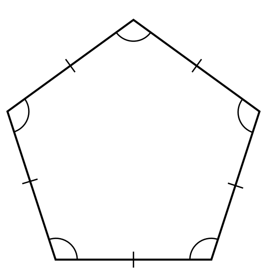
9. **Translation:** $(x, y) \rightarrow (x - 4, y - 4)$

Rotation: 270° about the origin

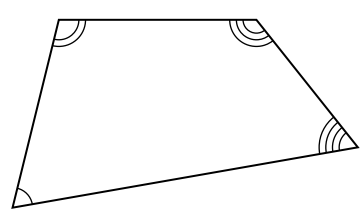


In Exercises 10 and 11, determine whether the figure has rotational symmetry. If so, describe any rotations that map the figure onto itself.

10.



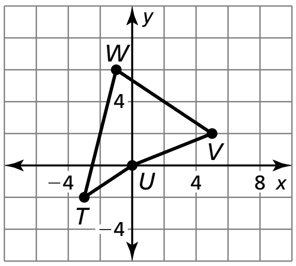
11.



Name____ Date__________

Practice B

1. Graph the polygon and its image after a $90°$ rotation about the origin.



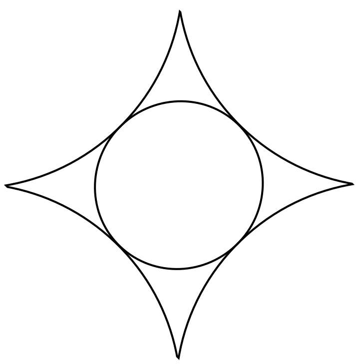
In Exercises 2 and 3, graph $\triangle CDE$ with vertices $C(-1, -3)$, $D(4, 2)$, and $E(-5, -1)$ and its image after the composition.

2. **Rotation:** $180°$ about the origin
Translation: $(x, y) \rightarrow (x + 3, y + 1)$

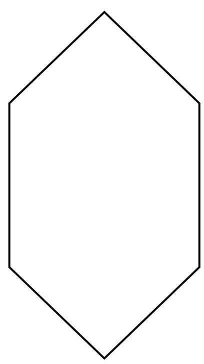
3. **Reflection:** in the line $x = y$
Rotation: $270°$ about the origin

In Exercises 4 and 5, determine whether the figure has rotational symmetry. If so, describe any rotations that map the figure onto itself.

4.



5.



6. Is it possible to have an object that does not have $360°$ of rotational symmetry? Explain your reasoning.

7. A figure that is rotated $60°$ is mapped back onto itself. Does the figure have rotational symmetry? Explain. How many times can you rotate the figure before it is back where it started?

8. Your friend claims that he can do a series of translations on any geometric object and get the same result as a rotation. Is your friend correct?

9. Your friend claims that she can do a series of reflections on any geometric object and get the same result as a rotation. Is your friend correct?

10. List the digits from 0−9 that have rotational symmetry, and state the angle of the symmetry.