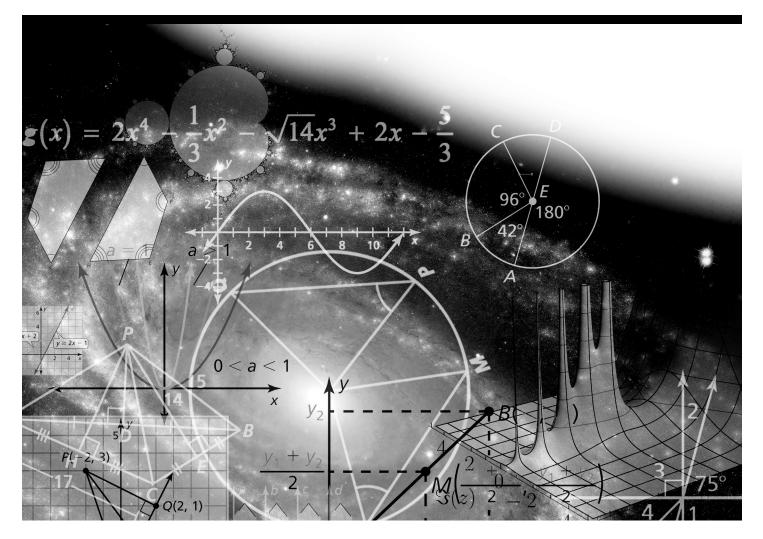
CHAPTER 7

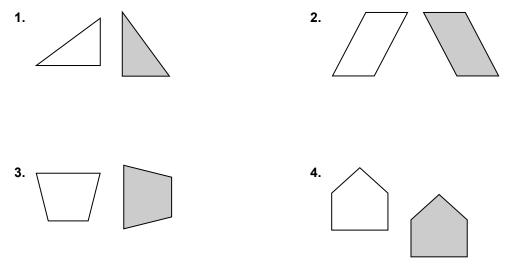
Transformations

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Chapter 7 Maintaining Mathematical Proficiency

Tell whether the shaded figure is a translation, reflection, or rotation of the nonshaded figure.



Find the perimeter and area of the polygon with the given vertices.

5. A(-4, 5), B(2, 5), C(2, -1), D(-4, -1)

6. E(0, 3), F(-3, 7), G(0, 7)

7.1 Translations For use with Exploration 7.1

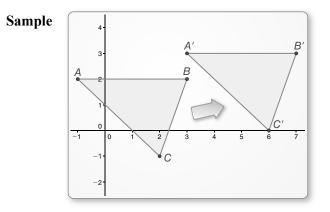
Essential Question How can you translate a figure in a coordinate plane?

EXPLORATION: Translating a Triangle in a Coordinate Plane

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

- **a.** Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.
- **b.** Copy the triangle and *translate* (or slide) it to form a new figure, called an *image*, $\Delta A'B'C'$. (read as "triangle A prime, B prime, C prime").
- **c.** What is the relationship between the coordinates of the vertices of $\triangle ABC$ and those of $\triangle A'B'C'$?
- **d.** What do you observe about the side lengths and angle measures of the two triangles?



EXPLORATION: Translating a Triangle in a Coordinate Plane

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

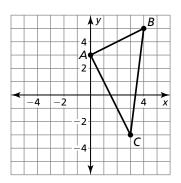
a. The point (x, y) is translated *a* units horizontally and *b* units vertically. Write a rule to determine the coordinates of the image of (x, y).

 $(x, y) \rightarrow (___, __]$

Name

7.1 Translations (continued)

- **b.** Use the rule you wrote in part (a) to translate $\triangle ABC$ 4 units left and 3 units down. What are the coordinates of the vertices of the image, $\Delta A'B'C'$?
- **c.** Draw $\triangle A'B'C'$. Are its side lengths the same as those of $\triangle ABC$? Justify your answer.



EXPLORATION: Comparing Angles of Translations 3

Work with a partner.

- **a.** In Exploration 2, is $\triangle ABC$ a right triangle? Justify your answer.
- **b.** In Exploration 2, is $\triangle A'B'C'$ a right triangle? Justify your answer.
- c. Do you think translations always preserve angle measures? Explain your reasoning.

Communicate Your Answer

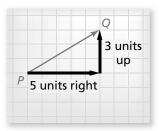
- 4. How can you translate a figure in a coordinate plane?
- **5.** In Exploration 2, translate $\triangle A'B'C'$ 3 units right and 4 units up. What are the coordinates of the vertices of the image, $\Delta A''B''C''$? How are these coordinates related to the coordinates of the vertices of the original triangle, $\triangle ABC$?



Core Concepts

Vectors

The diagram shows a vector. The **initial point**, or starting point, of the vector is *P*, and the **terminal point**, or ending point, is *Q*. The vector is named \overline{PQ} , which is read as "vector *PQ*." The **horizontal component** of \overline{PQ} is 5, and the **vertical component** is 3. The **component form** of a vector combines the horizontal and vertical components. So, the component form of \overline{PQ} is $\langle 5, 3 \rangle$.



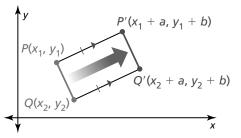
Notes:

Translations

A **translation** moves every point of a figure the same distance in the same direction. More specifically, a translation *maps*, or moves, the points *P* and *Q* of a plane figure along a vector $\langle a, b \rangle$ to the points *P'* and *Q'*, so that one of the following statements is true.

- PP' = QQ' and $\overline{PP'} \parallel \overline{QQ'}$, or
- PP' = QQ' and $\overline{PP'}$ and $\overline{QQ'}$ are collinear.

Notes:

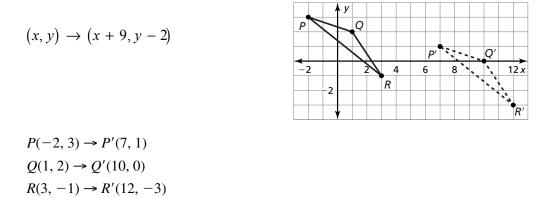


7.1 Practice (continued)

Worked-Out Examples

Example #1

Graph \triangle PQR with vertices P(-2, 3), Q(1, 2), and R(3, -1) and its image after the translation.



Example #2

Graph \triangle XYZ with vertices X(2, 4), Y(6, 0), and Z(7, 2) and its image after the composition.

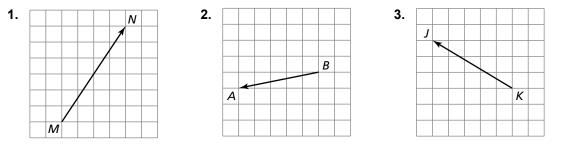
Translation: $(x, y) \rightarrow (x + 12, y + 4)$ Translation: $(x, y) \rightarrow (x - 5, y - 9)$

- 8	y						<i>X</i> ′			Ζ'	
- 4 -	X			Ζ					```	Υ'	
+			\leq	Ż					6		→ 0 x
		4	.	Yδ	5 👡		2	1	6	2	U X
-4		4		Y E	("'	1	$\overline{\mathbf{v}}$	Z"	6		

 $X(2, 4) \to X'(14, 8) \to X''(9, -1)$ $Y(6, 0) \to Y'(18, 4) \to Y''(13, -5)$ $Z(7, 2) \to Z'(19, 6) \to Z''(14, -3)$

Practice A

In Exercises 1–3, name the vector and write its component form.



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7.1 Practice (continued)

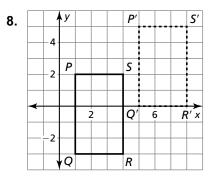
In Exercises 4–7, the vertices of $\triangle ABC$ are A(1, 2), B(5, 1), C(5, 4).

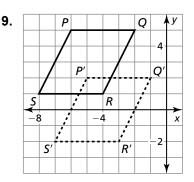
Translate $\triangle ABC$ using the given vector. Graph $\triangle ABC$ and its image.

- 4. ⟨-4, 0⟩
 5. ⟨-2, -4⟩
- 6. (0, -5)
- **7.** (1, -3)

				y			
			-4-				
_							.
-	_	_	_			_	
•	-4	1			4	1	x
<	-4	1	_1		4	1	x
-		1	-4-		2	1	x

In Exercises 8 and 9, write a rule for the translation of quadrilateral *PQRS* to quadrilateral P'Q'R'S'.





In Exercises 10 and 11, use the translation.

 $(x, y) \rightarrow (x + 6, y - 3)$

- **10.** What is the image of J(4, 5)?
- **11.** What is the image of R'(0, -5)?
- **12.** In a video game, you move a spaceship 1 unit left and 4 units up. Then, you move the spaceship 2 units left. Rewrite the composition as a single transformation.

F		-4 y		
		-2-		•
-4	-2		2	→ 4 x
		-2		
		-4		

Practice B

- **1.** The vertices of $\triangle FGH$ are F(-2, -6), G(3, 0), and H(1, -4). Translate $\triangle FGH$ using the vector $\langle -2, 7 \rangle$. Graph $\triangle FGH$ and its image.
- **2.** Find the component form of the vector that translates A(-4, 8) to A'(7, -9).
- **3.** Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$.

			-B-	y			
			Ā				
			2	C			
_	A						
<u> </u>	-4	1			<i>B'</i>	8	3x
			4		.• (<u>_</u> ′	
			-4	•			
			١	A			

In Exercises 4 and 5, use the translation $(x, y) \rightarrow (x - 4, y + 3)$.

- **4.** What is the image of G(-2, 4)?
- **5.** What is the image of H(-10, 5)?
- **6.** Graph $\triangle JKL$ with vertices J(-2, 8), K(1, -3), and L(5, 4) and its image after the composition.

Translation: $(x, y) \rightarrow (x + 6, y - 1)$ **Translation:** $(x, y) \rightarrow (x - 1, y - 7)$

- 7. Is the transformation given by $(x, y) \rightarrow (2x + 2, y + 1)$ a translation? Explain your reasoning.
- **8.** A popular kid's game has 15 tiles and 1 open space. The goal of the game is to rearrange the tiles to put them in order (from least to greatest, starting at the upper left-hand corner and going across each row). Use the figure to write the transformation(s) that describe the path of where the 8 tile is currently, and where it must be by the end of the game. Can this same translation be used to describe the path of all the tiles?

8	2	3	7
5	6	4	14
1	9		13
11	15	10	12

- **9.** Graph any triangle and translate it in any direction. Draw translation vectors for each vertex of the triangle. Is there a geometric relationship between all the translation vectors? Explain why this makes sense in terms of the slope of the line.
- **10.** Point P(4, -2) undergoes a translation given by $(x, y) \rightarrow (x + 3, x a)$, followed by another translation $(x, y) \rightarrow (x b, x + 7)$ to produce the image of P''(-5, 8). Find the values of a and b and point P'.