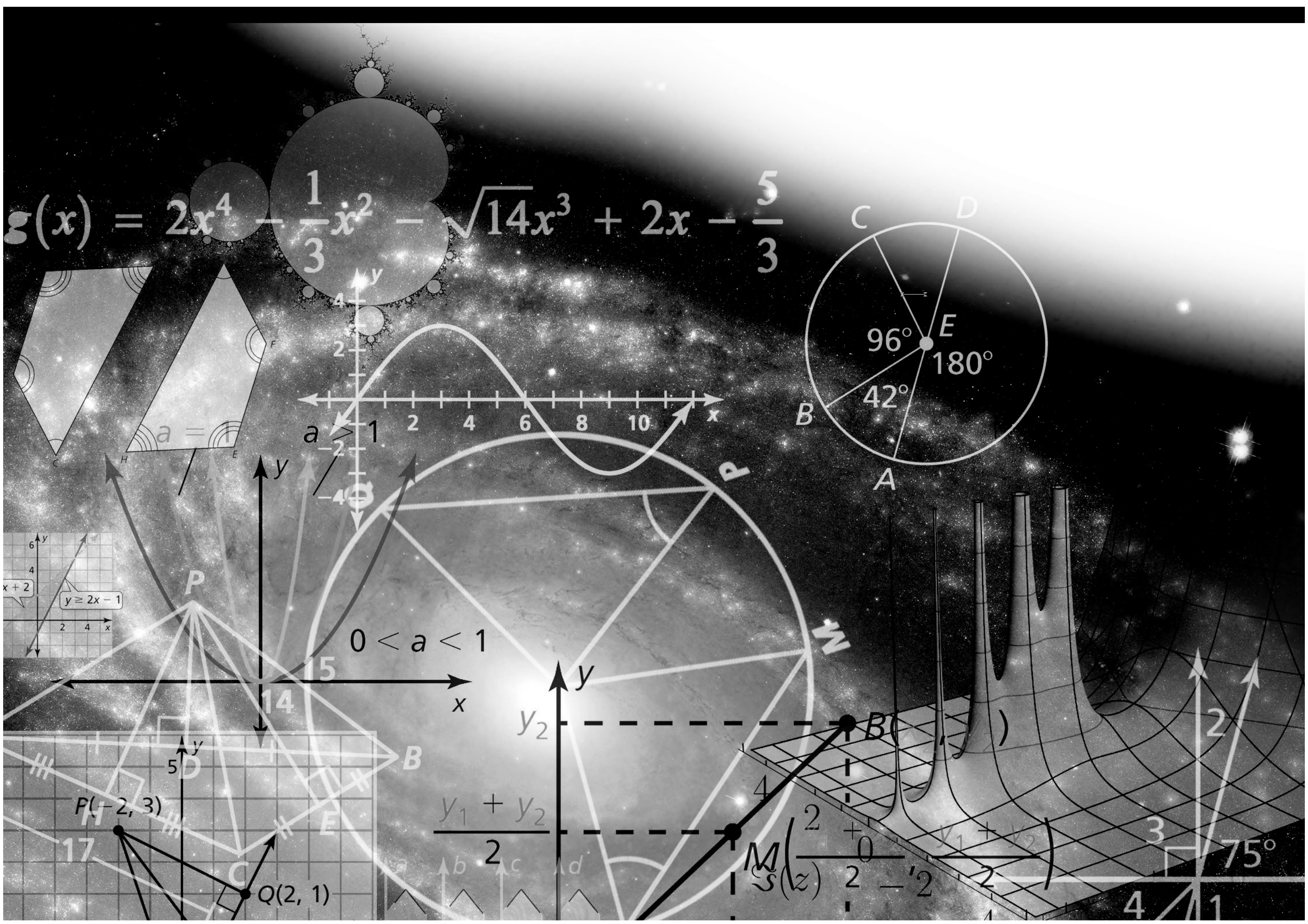
CHAPTER 5

Reasoning and Proofs



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Chapter 5 Maintaining Mathematical Proficiency

Write an equation for the *n*th term of the arithmetic sequence. Then find a_{20}.

1. 5, 11, 17, 23, …

2. 22, 34, 46, 58, …

3. −13, 0, 13, 26, …

4. −4.5, −4.0, −3.5, −3.0, …

5. 40, 25, 10, −5, …

6. $-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

Solve the equation.

7. $3x - 9 = 12$

8. $16 - 4y = 40$

9. $6z + 5 = 23$

10. $15 = 11q - 18$

11. $6r + 3 = 33$

12. $27 = 4s - 9$

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5.1 Conditional Statements
For use with Exploration 5.1

Essential Question
When is a conditional statement true or false?

A *conditional statement*, symbolized by $p \rightarrow q$, can be written as an "if-then statement" in which p is the *hypothesis* and q is the *conclusion*. Here is an example.

$$\textit{If } \underbrace{\textit{a polygon is a triangle}}_{\text{hypothesis, } p}, \textit{ then } \underbrace{\textit{the sum of its angle measures is } 180^\circ}_{\text{conclusion, } q}.$$

1 EXPLORATION: Determining Whether a Statement Is True or False

Work with a partner. A hypothesis can either be true or false. The same is true of a conclusion. For a conditional statement to be true, the hypothesis and conclusion do not necessarily both have to be true. Determine whether each conditional statement is true or false. Justify your answer.

a. If yesterday was Wednesday, then today is Thursday.

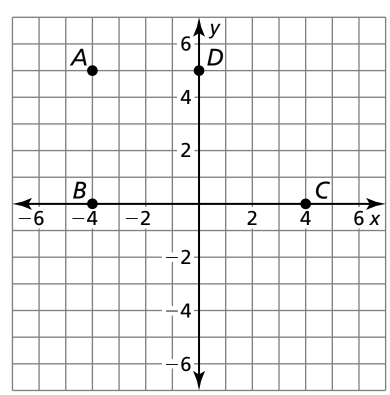
b. If an angle is acute, then it has a measure of 30°.

c. If a month has 30 days, then it is June.

d. If an even number is not divisible by 2, then 9 is a perfect cube.

2 EXPLORATION: Determining Whether a Statement Is True or False?

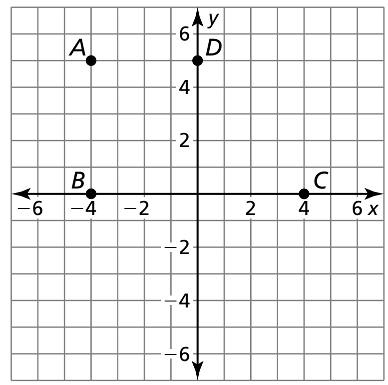
Work with a partner. Use the points in the coordinate plane to determine whether each statement is true or false. Justify your answer.



a. $\triangle ABC$ is a right triangle.

2 EXPLORATION: Determining Whether a Statement Is True or False (continued)

b. $\triangle BDC$ is an equilateral triangle.



c. $\triangle BDC$ is an isosceles triangle.

d. Quadrilateral *ABCD* is a trapezoid.

e. Quadrilateral *ABCD* is a parallelogram.

3 EXPLORATION: Determining Whether a Statement Is True or False

Work with a partner. Determine whether each conditional statement is true or false. Justify your answer.

a. If $\triangle ADC$ is a right triangle, then the Pythagorean Theorem is valid for $\triangle ADC$.

b. If $\angle A$ and $\angle B$ are complementary, then the sum of their measures is $180°$.

c. If figure *ABCD* is a quadrilateral, then the sum of its angle measures is $180°$.

d. If points *A*, *B*, and *C* are collinear, then they lie on the same line.

e. If $\overleftrightarrow{AB}$ and $\overleftrightarrow{BD}$ intersect at a point, then they form two pairs of vertical angles.

Communicate Your Answer

4. When is a conditional statement true or false?

5. Write one true conditional statement and one false conditional statement that are different from those given in Exploration 3. Justify your answer.

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5.1 Practice

For use after Lesson 5.1

Core Concepts

Conditional Statement

A **conditional statement** is a logical statement that has two parts, a *hypothesis* p and a *conclusion* q. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**.

Words If p, then q. **Symbols** $p \rightarrow q$ (read as "p implies q")

Notes:

Negation

The **negation** of a statement is the *opposite* of the original statement. To write the negation of a statement p, you write the symbol for negation $(\sim)$ before the letter. So, "not p" is written $\sim p$.

Words not p **Symbols** $\sim p$

Notes:

Related Conditionals

Consider the conditional statement below.

Words If p, then q. **Symbols** $p \rightarrow q$

Converse To write the **converse** of a conditional statement, exchange the hypothesis and the conclusion.

Words If q, then p. **Symbols** $q \rightarrow p$

Inverse To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion.

Words If not p, then not q. **Symbols** $\sim p \rightarrow \sim q$

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5.1 **Practice (continued)**

Related Conditionals (continued)

Contrapositive To write the **contrapositive** of a conditional statement, first write the converse. Then negate both the hypothesis and the conclusion.

Words If not q, then not p. **Symbols** $\sim q \rightarrow \sim p$

A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called **equivalent statements**.

Notes:

Biconditional Statement

When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

Words p if and only if q **Symbols** $p \leftrightarrow q$

Any definition can be written as a biconditional statement.

Notes:

Worked-Out Examples

Example #1

Copy the conditional statement. Identify the hypothesis and circle the conclusion.

If you like math, then you like science.

Hypothesis: You like math.

Conclusion: You like science.

Example #2

Rewrite the conditional statement in if-then form.

You are in a band, and you play the drums.

If you are in a band, then you play the drums.

Practice A

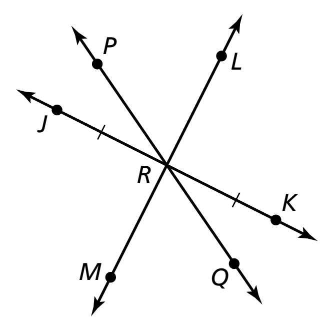
In Exercises 1 and 2, rewrite the conditional statement in if-then form.

1. $13x - 5 = -18$, because $x = -1$.

2. The sum of the measures of interior angles of a triangle is $180°$.

3. Let p be "Quadrilateral $ABCD$ is a rectangle" and let q be "the sum of the angle measures is $360°$." Write the conditional statement $p \rightarrow q$, the converse $q \rightarrow p$, the inverse $\sim p \rightarrow \sim q$, and the contrapositive $\sim q \rightarrow \sim p$ in words. Then decide whether each statement is true or false.

In Exercises 4–6, decide whether the statement about the diagram is true. Explain your answer using the definitions you have learned.



4. $\overline{LM}$ bisects $\overline{JK}$

5. $\angle JRP$ and $\angle PRL$ are complementary.

6. $\angle MRQ \cong \angle PRL$

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Practice B

In Exercises 1 and 2, copy the conditional statement. Underline the hypothesis and circle the conclusion.

1. If you like to eat, then you are a good cook.

2. If an animal is a bear, then it is a mammal.

3. Let p be "a tree is an oak tree" and let q be "it is a deciduous tree." Write each statement in words. Then decide whether it is true or false.

   a. the conditional statement $p \rightarrow q$

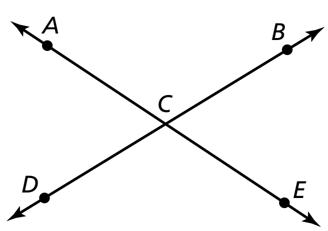
   b. the converse $q \rightarrow p$

   c. the inverse $\sim p \rightarrow \sim q$

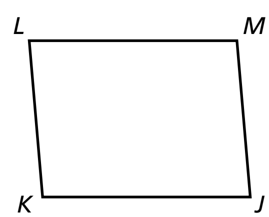
   d. the contrapositive $\sim q \rightarrow \sim p$

In Exercises 4 and 5, decide whether the statement about the diagram is true. Explain your answer using the definitions you have learned.

4. $\angle ACB$ and $\angle DCE$ are vertical angles.



5. $\overline{KL} \perp \overline{LM}$



6. Rewrite the two statements as a single biconditional statement: A rectangle is a quadrilateral that has all perpendicular sides. If all sides of a quadrilateral are perpendicular, then it is a rectangle.

7. Your friend claims that only true conditional statements have a true contrapositive. Is your friend correct? Explain your reasoning.

8. Rewrite the conditional statement in if-then form: $3x + 2 = 23$, because $x = 7$.

9. Write a series of if-then statements that allow you to find the measure of each angle, given that $\angle ILH = 38°$. Use the definitions of supplementary and complementary angles that you have learned so far.