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3.6 Graphing Rational Functions

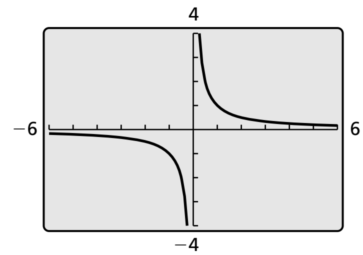
For use with Exploration 3.6

Essential Question What are some of the characteristics of the graph of a rational function?

The parent function for rational functions with a linear numerator and a linear denominator is

$$f(x) = \frac{1}{x}. \qquad \text{Parent function}$$

The graph of this function, shown at the right, is a *hyperbola*.



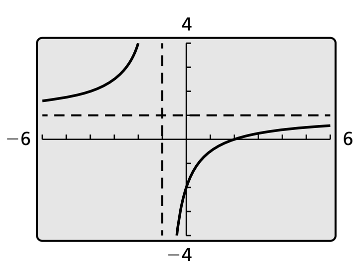
1 EXPLORATION: Identifying Graphs of Rational Functions

Work with a partner. Each function is a transformation of the graph of the parent function $f(x) = \frac{1}{x}$. Match the function with its graph. Explain your reasoning. Then describe the transformation.

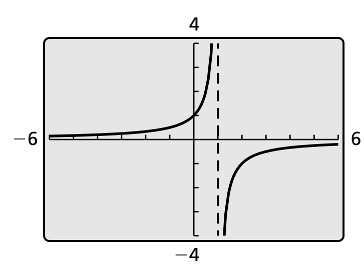
a. $g(x) = \frac{1}{x - 1}$ **b.** $g(x) = \frac{-1}{x - 1}$ **c.** $g(x) = \frac{x + 1}{x - 1}$

d. $g(x) = \frac{x - 2}{x + 1}$ **e.** $g(x) = \frac{x}{x + 2}$ **f.** $g(x) = \frac{-x}{x + 2}$

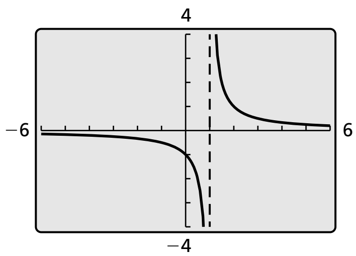
A.

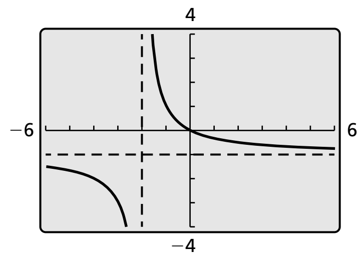


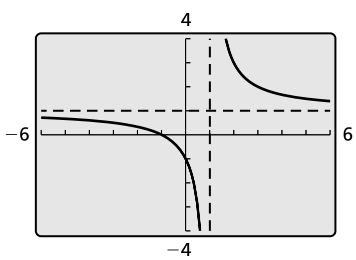
B.



1 EXPLORATION: Identifying Graphs of Rational Functions (continued)

C.


D.


E.


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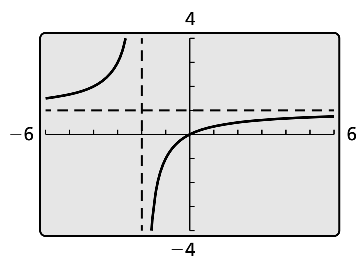

Communicate Your Answer

2. What are some of the characteristics of the graph of a rational function?

3. Determine the intercepts, asymptotes, domain, and range of the rational function $g(x) = \dfrac{x - a}{x - b}$.

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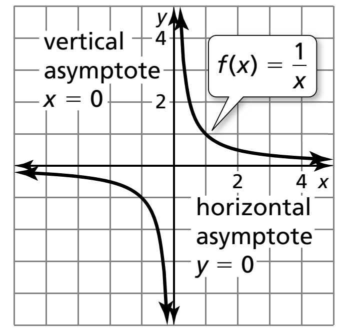
3.6 Practice
For use after Lesson 3.6

Core Concepts

Parent Function for Simple Rational Functions

The graph of the parent function $f(x) = \dfrac{1}{x}$ is a *hyperbola*, which consists of two symmetrical parts called branches. The domain and range are all nonzero real numbers.



Any function of the form $g(x) = \dfrac{a}{x}\ (a \neq 0)$ has the same asymptotes, domain, and range as the function $f(x) = \dfrac{1}{x}$.

Notes:

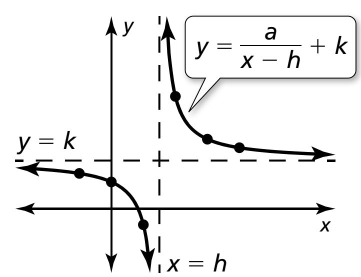
Graphing Translations of Simple Rational Functions

To graph a rational function of the form $y = \dfrac{a}{x - h} + k$, follow these steps:

Step 1 Draw the asymptotes $x = h$ and $y = k$.

Step 2 Plot points to the left and to the right of the vertical asymptote.

Step 3 Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



Notes:

Worked-Out Examples

Example #1

Graph the function. State the domain and range.

$y = \dfrac{1}{x + 2}$

Step 1 Draw the asymptotes $x = -2$ and $y = 0$.

Step 2 Plot points to the left of the vertical asymptote, such as $\left(-5, -\frac{1}{3}\right)$, $\left(-4, -\frac{1}{2}\right)$, and $(-3, -1)$. Plot points to the right of the vertical asymptote, such as $(-1, 1)$, $\left(0, \frac{1}{2}\right)$, and $\left(1, \frac{1}{3}\right)$.

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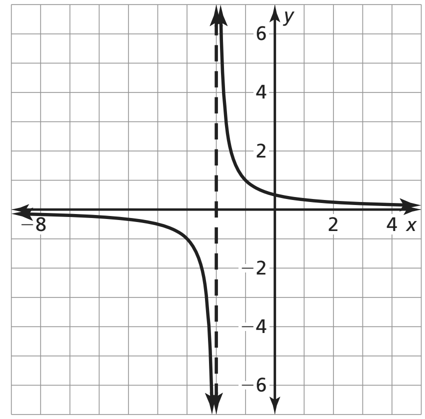
Step 3 Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



The domain is all real numbers except −2 and the range is all real numbers except 0.

Example #2

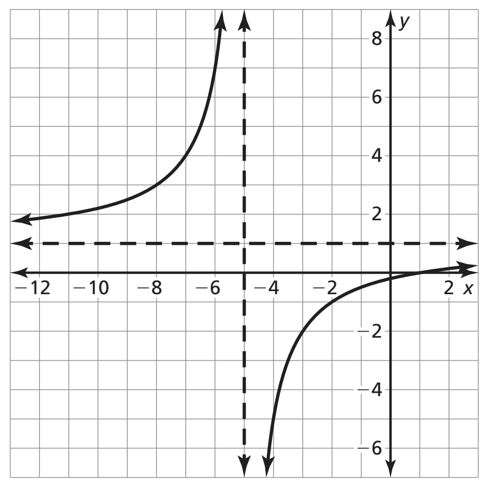
Graph the function. State the domain and range.

$$y = \frac{x - 1}{x + 5}$$

Step 1 Draw the asymptotes. Solve $x + 5 = 0$ for x to find the vertical asymptote $x = -5$. The horizontal asymptote is the line $y = \frac{a}{c} = \frac{1}{1} = 1$.

Step 2 Plot points to the left of the vertical asymptote, such as (−8, 3), (−7, 4), and (−6, 7). Plot points to the right of the vertical asymptote, such as (−4, −5), (−3, −2), and (−2, −1).

Step 3 Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



The domain is all real numbers except −5 and the range is all real numbers except 1.

Practice A

In Exercises 1 and 2, graph the function. Compare the graph with the graph of $f(x) = \frac{1}{x}$.

1. $g(x) = \frac{0.25}{x}$

2. $h(x) = \frac{-2}{x}$

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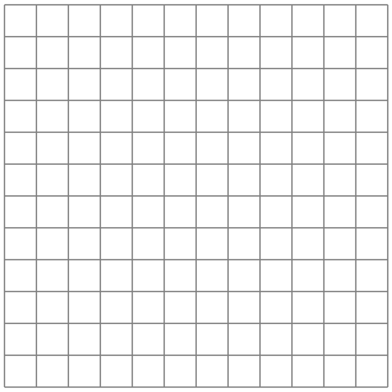
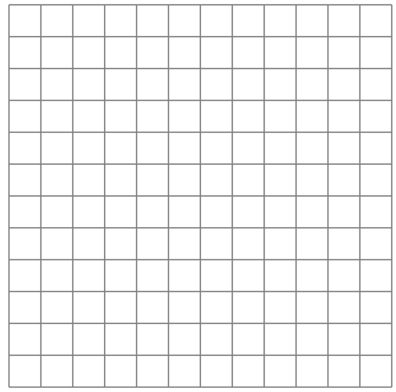
3.6 Practice (continued)

In Exercises 3 and 4, graph the function. State the domain and range.

3. $k(x) = \dfrac{1}{x - 3} + 5$

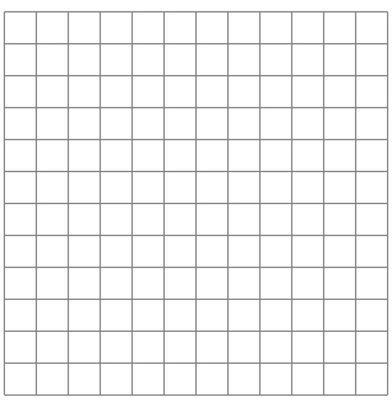


4. $m(x) = \dfrac{-3}{x} - 4$

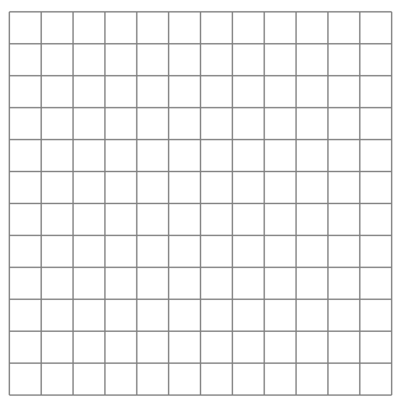


In Exercises 5 and 6, rewrite the function in the form $g(x) = \dfrac{a}{x - h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \dfrac{a}{x}$.

5. $g(x) = \dfrac{x + 2}{x - 5}$



6. $g(x) = \dfrac{2x + 8}{3x - 12}$



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Practice B

In Exercises 1–3, graph the function. Compare the graph with the graph of $f(x) = \frac{1}{x}$.

1. $h(x) = \frac{12}{x}$

2. $g(x) = \frac{-8}{x}$

3. $h(x) = \frac{0.2}{x}$

In Exercises 4–15, graph the function. State the domain and range.

4. $f(x) = \frac{5}{x} - 2$

5. $g(x) = \frac{3}{x + 4}$

6. $y = \frac{-8}{x - 3}$

7. $h(x) = \frac{-1}{x + 5}$

8. $y = \frac{-2}{x + 1} + 3$

9. $y = \frac{9}{x - 4} - 2$

10. $f(x) = \frac{x + 5}{x - 4}$

11. $g(x) = \frac{x - 3}{2x + 8}$

12. $h(x) = \frac{-8x + 3}{5x + 2}$

13. $y = \frac{3x - 1}{5x - 1}$

14. $y = \frac{-3x}{-4x - 1}$

15. $y = \frac{-2x + 5}{-x + 8}$

In Exercises 16–21, rewrite the function in the form $g(x) = \frac{a}{x - h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \frac{a}{x}$.

16. $g(x) = \frac{3x + 7}{x + 2}$

17. $g(x) = \frac{4x - 2}{x - 3}$

18. $g(x) = \frac{4x - 10}{x + 5}$

19. $g(x) = \frac{x + 12}{x - 3}$

20. $g(x) = \frac{5x - 30}{x + 4}$

21. $g(x) = \frac{7x - 2}{x + 6}$

22. You are creating statues made of cement. The mold costs \$300. The material for each statue costs \$22.

    a. Estimate how many statues must be made for the average cost per statue to fall below \$30.

    b. What happens to the average cost as more statues are created?

23. The concentration c of a certain drug in a patient's bloodstream t hours after an injection is given by $c(t) = \frac{t}{4t^2 + 1}$.

    a. Use a graphing calculator to graph the function. Describe a reasonable domain and range.

    b. Determine the time at which the concentration is the highest.