Name ______________________________ Date __________

3.4 Solving Radical Equations and Inequalities

For use with Exploration 3.4

Essential Question How can you solve a radical equation?

1 EXPLORATION: Solving Radical Equations

Work with a partner. Match each radical equation with the graph of its related radical function. Explain your reasoning. Then use the graph to solve the equation, if possible. Check your solutions.

a. $\sqrt{x-1} - 1 = 0$

b. $\sqrt{2x+2} - \sqrt{x+4} = 0$

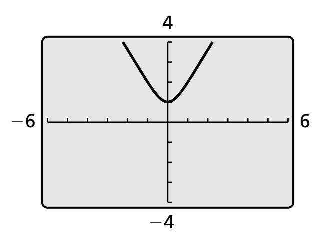
c. $\sqrt{9-x^2} = 0$

d. $\sqrt{x+2} - x = 0$

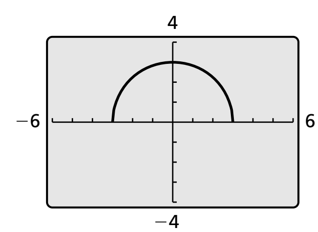
e. $\sqrt{-x+2} - x = 0$

f. $\sqrt{3x^2+1} = 0$

A.



B.



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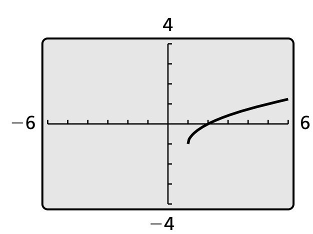

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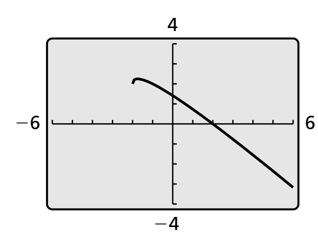
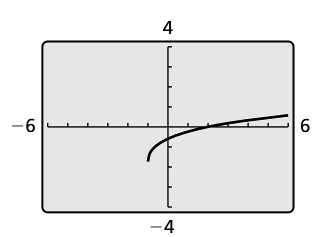
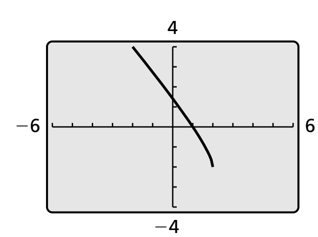


E.



F.



2 EXPLORATION: Solving Radical Equations

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Look back at the radical equations in Exploration 1. Suppose that you did not know how to solve the equations using a graphical approach.

a. Show how you could use a *numerical approach* to solve one of the equations. For instance, you might use a spreadsheet to create a table of values.

b. Show how you could use an *analytical approach* to solve one of the equations. For instance, look at the similarities between the equations in Exploration 1. What first step may be necessary so you could square each side to eliminate the radical(s)? How would you proceed to find the solution?

Communicate Your Answer

3. How can you solve a radical equation?

4. Would you prefer to use a graphical, numerical, or analytical approach to solve the given equation? Explain your reasoning. Then solve the equation.

$$\sqrt{x+3} - \sqrt{x-2} = 1$$

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3.4 Practice

For use after Lesson 3.4

Core Concepts

Solving Radical Equations

To solve a radical equation, follow these steps:

Step 1 Isolate the radical on one side of the equation, if necessary.

Step 2 Raise each side of the equation to the same exponent to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

Step 3 Solve the resulting equation using techniques you learned previously. Check your solution.

Notes:

Worked-Out Examples

Example #1

Solve the equation. Check your solution.

$$\sqrt[3]{x} - 10 = -7$$
$$\sqrt[3]{x} = 3$$
$$\left(\sqrt[3]{x}\right)^3 = 3^3$$
$$x = 27$$

Check: $\sqrt[3]{27} - 10 \stackrel{?}{=} -7$

$$3 - 10 \stackrel{?}{=} -7$$
$$-7 = -7 \checkmark$$

The solution is $x = 27$.

Example #2

Solve the equation. Check your solution(s).

$$2x^{3/4} - 14 = 40$$
$$2x^{3/4} = 54$$
$$x^{3/4} = 27$$
$$(x^{3/4})^{4/3} = 27^{4/3}$$
$$x = 81$$

Check:

$$2(81)^{3/4} - 14 \stackrel{?}{=} 40$$
$$2(27) - 14 \stackrel{?}{=} 40$$
$$40 = 40 \checkmark$$

The solution is $x = 81$.

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3.4 **Practice** (continued)

Practice A

In Exercises 1–10, solve the equation. Check your solution(s).

1. $\sqrt{1 - x} = 7$

2. $\sqrt[3]{5x + 1} = -4$

3. $\frac{1}{4}\sqrt[4]{2x} + 6 = 10$

4. $2\sqrt[3]{13x - 5} = 10$

5. $x - 7 = \sqrt{x - 5}$

6. $\sqrt[3]{486 - 27x^3} = 3x$

7. $4\sqrt{x + 1} = x + 1$

8. $\sqrt{2x + 2} - 3\sqrt{x + 1} = 0$

9. $2 - \sqrt[4]{2x - 6} = 14$

10. $\sqrt{x + 7} + 2 = \sqrt{3 - x}$

3.4 Practice (continued)

In Exercises 11 and 12, solve the equation. Check your solution(s).

11. $\frac{1}{2}x^{5/2} = 16$

12. $(6x + 10)^{7/3} + 28 = 156$

In Exercises 13–15, solve the inequality.

13. $-4\sqrt{x - 1} + 3 \geq -1$

14. $\sqrt[3]{\frac{2}{3}x + 1} < 6$

15. $2\sqrt{\frac{3}{4}x} - 39 \leq -25$

16. In basketball, the term "hang time" is the amount of time that a player is suspended in the air when making a basket. To win a slam-dunk contest, players try to maximize their hang time. A player's hang time is given by the equation $t = 0.5\sqrt{h}$, where t is the time (in seconds) and h is the height (in feet) of the jump. The second-place finisher of a slam-dunk contest had a hang time of 1 second, and the winner had a hang time of 1.2 seconds. How many feet higher did the winner jump than the second-place finisher?

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Practice B

In Exercises 1–6, solve the equation. Check your solution.

1. $\sqrt[3]{x - 14} = -2$

2. $-5\sqrt{16x} + 17 = -8$

3. $\frac{1}{4}\sqrt[3]{2x} + 8 = 6$

4. $\sqrt{3x} - \frac{3}{4} = 0$

5. $3\sqrt[5]{x} + 9 = 15$

6. $\sqrt[4]{8x} - 16 = -12$

In Exercises 7–12, solve the equation. Check your solution(s).

7. $\sqrt{10x + 24} = x + 12$

8. $x + 3 = \sqrt{\frac{22}{3}x + 9}$

9. $\sqrt[4]{2 - 25x^2} = 5x$

10. $\sqrt{4x - 4} - \sqrt{x + 8} = 0$

11. $\sqrt[3]{4x - 1} = \sqrt[3]{6x + 5}$

12. $\sqrt{4x - 10} = \sqrt{2x - 13} + 1$

In Exercises 13–15, solve the equation. Check your solution(s).

13. $3x^{2/3} - 30 = 18$

14. $(6x + 8)^{1/2} - 3x = 0$

15. $(2x^2 + 8)^{1/4} = x$

In Exercises 16–18, solve the inequality.

16. $4\sqrt{x} + 3 \le 23$

17. $\sqrt{x + 10} \ge 6$

18. $-3\sqrt{x + 2} < 15$

19. "Hang time" is the time you are suspended in the air during a jump. Your hang time t in seconds is given by the function $t = 0.5\sqrt{h}$, where h is the height (in feet) of the jump. A kite sailor has a hang time of 2.5 seconds. Find the height of the kite sailor's jump.

In Exercises 20–23, solve the nonlinear system. Justify your answer with a graph.

20. $y^2 = x + 2$
$y = x + 2$

21. $y^2 = -x + 7$
$y = x - 1$

22. $x^2 + y^2 = 9$
$y = x - 3$

23. $x^2 + y^2 = 16$
$y = x + 4$

24. The speed s (in miles per hour) of a car can be given by $s = \sqrt{30fd}$, where f is the coefficient of friction and d is the stopping distance (in feet). The coefficient of friction for a snowy road is 0.30. You are driving 20 miles per hour and approaching an intersection. How far away from the intersection must you begin to brake?