Name____ Date__________

3.3 Graphing Radical Functions
For use with Exploration 3.3

Essential Question How can you identify the domain and range of a radical function?

1 EXPLORATION: Identifying Graphs of Radical Functions

Work with a partner. Match each function with its graph. Explain your reasoning. Then identify the domain and range of each function.

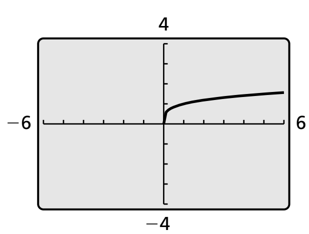
a. $f(x) = \sqrt{x}$

b. $f(x) = \sqrt[3]{x}$

c. $f(x) = \sqrt[4]{x}$

d. $f(x) = \sqrt[5]{x}$

A.



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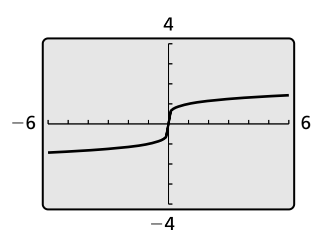

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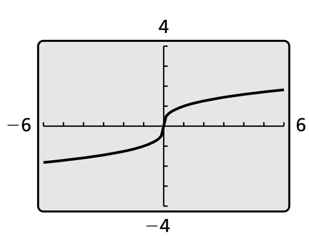

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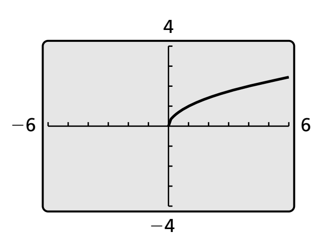


Name ____ Date _________

2 EXPLORATION: Identifying Graphs of Transformations

Work with a partner. Match each transformation of $f(x) = \sqrt{x}$ with its graph. Explain your reasoning. Then identify the domain and range of each function.

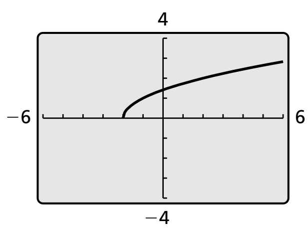
a. $g(x) = \sqrt{x + 2}$

b. $g(x) = \sqrt{x - 2}$

c. $g(x) = \sqrt{x + 2} - 2$

d. $g(x) = -\sqrt{x + 2}$

A.



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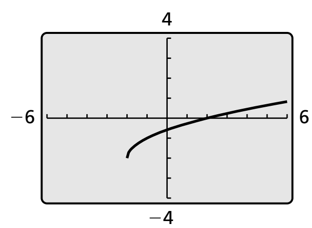

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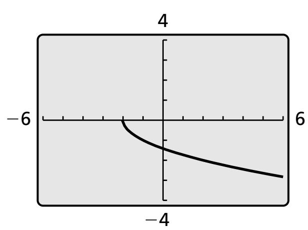

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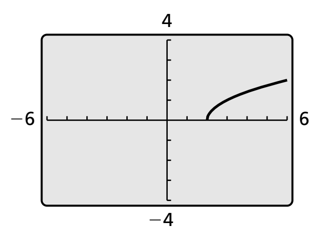


Communicate Your Answer

3. How can you identify the domain and range of a radical function?

4. Use the results of Exploration 1 to describe how the domain and range of a radical function are related to the index of the radical.

Name____ Date__________

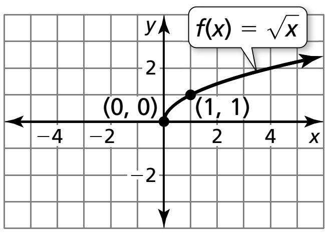
3.3 Practice

For use after Lesson 3.3

Core Concepts

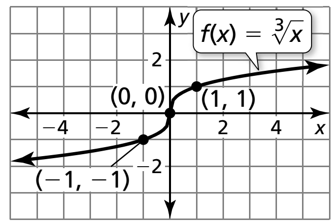
Parent Functions for Square Root and Cube Root Functions

The parent function for the family of square root functions is $f(x) = \sqrt{x}$.



Domain: $x \geq 0$, Range: $y \geq 0$

The parent function for the family of cube root functions is $f(x) = \sqrt[3]{x}$.



Domain and range: All real numbers

Notes:

Transformation	$f(x)$ Notation	Examples	
Horizontal Translation Graph shifts left or right.	$f(x-h)$	$g(x) = \sqrt{x-2}$ $g(x) = \sqrt{x+3}$	2 units right 3 units left
Vertical Translation Graph shifts up or down.	$f(x) + k$	$g(x) = \sqrt{x} + 7$ $g(x) = \sqrt{x} - 1$	7 units up 1 unit down
Reflection Graph flips over x- or y-axis.	$f(-x)$ $-f(x)$	$g(x) = \sqrt{-x}$ $g(x) = -\sqrt{x}$	in the y-axis in the x-axis
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y-axis.	$f(ax)$	$g(x) = \sqrt{3x}$ $g(x) = \sqrt{\frac{1}{2}x}$	shrink by a factor of $\frac{1}{3}$ stretch by a factor of 2
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x-axis.	$a \bullet f(x)$	$g(x) = 4\sqrt{x}$ $g(x) = \frac{1}{5}\sqrt{x}$	stretch by a factor of 4 shrink by a factor of $\frac{1}{5}$

Notes:

Name ____ Date __________

3.3 Practice (continued)

Worked-Out Examples

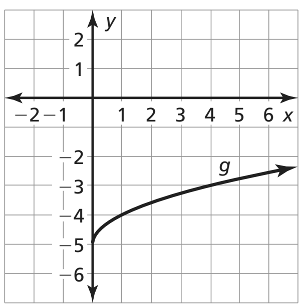
Example #1

Graph the function. Identify the domain and range of the function.

$g(x) = \sqrt{x} - 5$

Make a table of values and sketch the graph.

x	0	1	2	3	4
y	−5	−4	−3.59	−3.27	−3



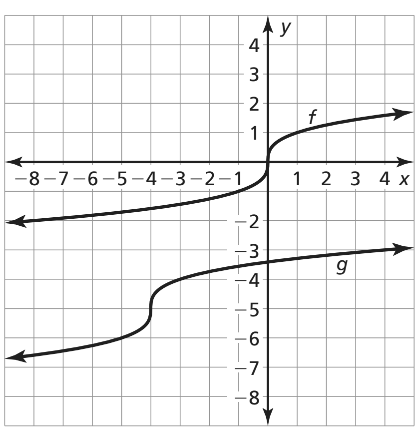
The radicand of a square root must be nonnegative. So, the domain is $x \geq 0$. The range is $y \geq -5$.

Example #2

Describe the transformation of f represented by g. Then graph each function.

$f(x) = \sqrt[3]{x}, \; g(x) = \sqrt[3]{x + 4} - 5$

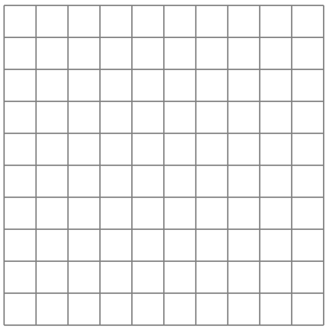
Notice that the function is of the form $g(x) = \sqrt[3]{x - h} + k$, where $h = -4$ and $k = -5$. So, the graph of g is a translation 4 units left and 5 units down of the graph of f.



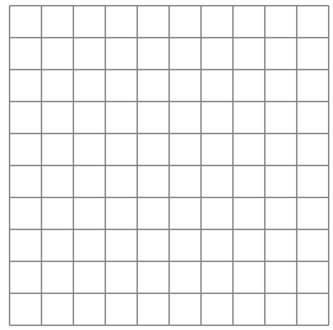
Practice A

In Exercises 1 and 2, graph the function. Identify the domain and range of each function.

1. $f(x) = \sqrt[3]{-3x} + 1$



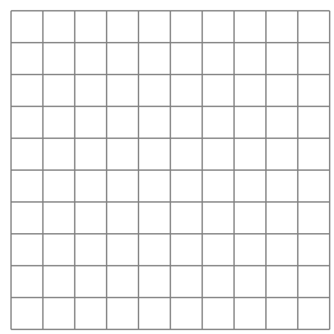
2. $g(x) = 2(x - 5)^{1/2} - 4$



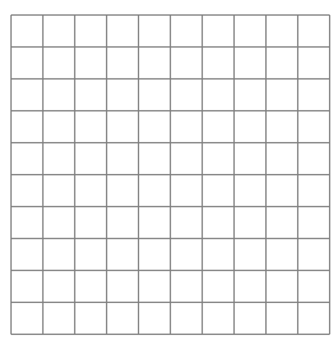
3. Describe the transformation of $f(x) = \sqrt[4]{2x} + 5$ represented by $g(x) = -\sqrt[4]{2x} - 5$.

4. Write a rule for g if g is a horizontal shrink by a factor of $\frac{5}{6}$, followed by a translation 10 units to the left of the graph of $f(x) = \sqrt[3]{15x + 1}$.

5. Use a graphing calculator to graph $8x = y^2 + 5$. Identify the vertex and the direction that the parabola opens.



6. Use a graphing calculator to graph $x^2 = 49 - y^2$. Identify the center, radius, and intercepts of the circle.



Name____ Date__________

Practice B

In Exercises 1–6, graph the function. Identify the domain and range of the function.

1. $g(x) = -\sqrt{x} + 2$ **2.** $f(x) = \sqrt[3]{-4x}$ **3.** $f(x) = \frac{1}{4}\sqrt{x + 5}$

4. $h(x) = (5x)^{1/2} - 2$ **5.** $g(x) = -2(x - 3)^{1/3}$ **6.** $h(x) = -\sqrt[5]{x}$

In Exercises 7–12, describe the transformation of *f* represented by *g*. Then graph each function.

7. $f(x) = \sqrt{x}; g(x) = 4\sqrt{x - 2}$ **8.** $f(x) = \sqrt[3]{x}; g(x) = \sqrt[3]{x - 5} - 1$

9. $f(x) = x^{1/4}; g(x) = \frac{1}{3}(-x)^{1/4}$ **10.** $f(x) = x^{1/3}; g(x) = \frac{1}{2}x^{1/3} - 3$

11. $f(x) = \sqrt[4]{x}; g(x) = -\sqrt[4]{x - 1} + 3$ **12.** $f(x) = \sqrt[5]{x}; g(x) = \sqrt[5]{-243x} - 2$

In Exercises 13–15, use a graphing calculator to graph the function. Then identify the domain and range of the function.

13. $g(x) = \sqrt[3]{2x^2 - 3x}$ **14.** $f(x) = \sqrt{\frac{1}{3}x^2 - x + 2}$ **15.** $h(x) = \sqrt[3]{3x^2 - 6x + 2}$

In Exercises 16 and 17, write a rule for *g* described by the transformations of the graph of *f*.

16. Let g be a horizontal stretch by a factor of 2, followed by a translation 2 units up of the graph of $f(x) = \sqrt{3x}$.

17. Let g be a translation 1 unit up and 4 units left, followed by a reflection in the y-axis of the graph of $f(x) = \sqrt{-x} - \frac{1}{2}$.

In Exercises 18 and 19, use a graphing calculator to graph the equation of the parabola. Identify the vertex and the direction that the parabola opens.

18. $3y^2 + 5 = x$ **19.** $x - 3 = -\frac{1}{2}y^2$

In Exercises 20 and 21, use a graphing calculator to graph the equation of the circle. Identify the center, radius, and intercepts.

20. $y^2 = 81 - (x + 3)^2$ **21.** $x^2 + y^2 + 8y + 15 = 0$