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3.2 Properties of Rational Exponents and Radicals

For use with Exploration 3.2

Essential Question How can you use properties of exponents to simplify products and quotients of radicals?

1 EXPLORATION: Reviewing Properties of Exponents

Work with a partner. Let *a* and *b* be real numbers. Use the properties of exponents to complete each statement. Then match each completed statement with the property it illustrates.

Statement	Property
a. $a^{-2} = \_\_\_\_\_, \; a \neq 0$	**A.** Product of Powers
b. $(ab)^4 = \_\_\_\_\_\_$	**B.** Power of a Power
c. $(a^3)^4 = \_\_\_\_\_\_$	**C.** Power of a Product
d. $a^3 \bullet a^4 = \_\_\_\_\_\_$	**D.** Negative Exponent
e. $\left(\frac{a}{b}\right)^3 = \_\_\_\_\_\_, \; b \neq 0$	**E.** Zero Exponent
f. $\frac{a^6}{a^2} = \_\_\_\_\_\_, \; a \neq 0$	**F.** Quotient of Powers
g. $a^0 = \_\_\_\_\_\_, \; a \neq 0$	**G.** Power of a Quotient

2 EXPLORATION: Simplifying Expressions with Rational Exponents

Work with a partner. Show that you can apply the properties of integer exponents to rational exponents by simplifying each expression. Use a calculator to check your answers.

a. $5^{2/3} \bullet 5^{4/3}$

b. $3^{1/5} \bullet 3^{4/5}$

c. $(4^{2/3})^3$

d. $(10^{1/2})^4$

e. $\frac{8^{5/2}}{8^{1/2}}$

f. $\frac{7^{2/3}}{7^{5/3}}$

3 EXPLORATION: Simplifying Products and Quotients of Radicals

Work with a partner. Use the properties of exponents to write each expression as a single radical. Then evaluate each expression. Use a calculator to check your answers.

a. $\sqrt{3} \bullet \sqrt{12}$ **b.** $\sqrt[3]{5} \bullet \sqrt[3]{25}$ **c.** $\sqrt[4]{27} \bullet \sqrt[4]{3}$

d. $\dfrac{\sqrt{98}}{\sqrt{2}}$ **e.** $\dfrac{\sqrt[4]{4}}{\sqrt[4]{1024}}$ **f.** $\dfrac{\sqrt[3]{625}}{\sqrt[3]{5}}$

Communicate Your Answer

4. How can you use properties of exponents to simplify products and quotients of radicals?

5. Simplify each expression.

a. $\sqrt{27} \bullet \sqrt{6}$ **b.** $\dfrac{\sqrt[3]{240}}{\sqrt[3]{15}}$ **c.** $\left(5^{1/2} \bullet 16^{1/4}\right)^2$

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3.2 Practice
For use after Lesson 3.2

Core Concepts

Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers, such that the quantities in each property are real numbers.

Property Name	Definition	Example
Product of Powers	$a^m \bullet a^n = a^{m+n}$	$5^{1/2} \bullet 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \bullet 2)} = 3^5 = 243$
Power of a Product	$(ab)^m = a^m b^m$	$(16 \bullet 9)^{1/2} = 16^{1/2} \bullet 9^{1/2} = 4 \bullet 3 = 12$
Negative Exponent	$a^{-m} = \dfrac{1}{a^m}, a \neq 0$	$36^{-1/2} = \dfrac{1}{36^{1/2}} = \dfrac{1}{6}$
Zero Exponent	$a^0 = 1, a \neq 0$	$213^0 = 1$
Quotient of Powers	$\dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\dfrac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
Power of a Quotient	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}, b \neq 0$	$\left(\dfrac{27}{64}\right)^{1/3} = \dfrac{27^{1/3}}{64^{1/3}} = \dfrac{3}{4}$

Notes:

Properties of Radicals

Let a and b be real numbers and let n be an integer greater than 1.

Property Name	Definition	Example
Product Property	$\sqrt[n]{a \bullet b} = \sqrt[n]{a} \bullet \sqrt[n]{b}$	$\sqrt[3]{4} \bullet \sqrt[3]{2} = \sqrt[3]{8} = 2$
Quotient Property	$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$	$\dfrac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\dfrac{162}{2}} = \sqrt[4]{81} = 3$

Notes:

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3.2 **Practice (continued)**

Worked-Out Examples

Example #1

With the expression in simplest form.

$$\frac{1}{2+\sqrt{5}} = \frac{1}{2+\sqrt{5}} \cdot \frac{2-\sqrt{5}}{2-\sqrt{5}}$$
$$= \frac{1\left(2-\sqrt{5}\right)}{2^2-\left(\sqrt{5}\right)^2}$$
$$= \frac{2-\sqrt{5}}{4-5}$$
$$= \frac{2-\sqrt{5}}{-1}$$
$$= \sqrt{5}-2$$

Example #2

Simplify the expression.

$$5\sqrt{12} - 19\sqrt{3} = 5\sqrt{4 \cdot 3} - 19\sqrt{3}$$
$$= 5\sqrt{4}\sqrt{3} - 19\sqrt{3}$$
$$= 10\sqrt{3} - 19\sqrt{3}$$
$$= (10-19)\sqrt{3}$$
$$= -9\sqrt{3}$$

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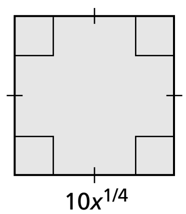
3.2 Practice (continued)

Practice A

In Exercises 1–4, use the properties of rational exponents to simplify the expression.

1. $\left(2^3 \bullet 3^3\right)^{-1/3}$ **2.** $\dfrac{10}{10^{-4/5}}$ **3.** $\left(\dfrac{52^5}{4^5}\right)^{1/6}$ **4.** $\dfrac{3^{1/3} \bullet 27^{2/3}}{8^{4/3}}$

5. Find simplified expressions for the perimeter and area of the given figure.



In Exercises 6–8, use the properties of radicals to simplify the expression.

6. $\sqrt[6]{25} \bullet \sqrt[6]{625}$ **7.** $\dfrac{\sqrt{343}}{\sqrt{7}}$ **8.** $\dfrac{\sqrt[3]{25} \bullet \sqrt[3]{10}}{\sqrt[3]{2}}$

In Exercises 9–12, write the expression in simplest form.

9. $\sqrt[7]{384}$ **10.** $\sqrt[3]{\dfrac{5}{9}}$

11. $\dfrac{1}{4 - \sqrt{5}}$ **12.** $\dfrac{\sqrt{2}}{1 + \sqrt{6}}$

In Exercises 13–16, write the expression in simplest form. Assume all variables are positive.

13. $-2\sqrt[3]{5} + 40\sqrt[3]{5}$ **14.** $2(1250)^{1/4} - 5(32)^{1/4}$

15. $\dfrac{\sqrt[4]{x} \bullet \sqrt[4]{81x}}{\sqrt[4]{16x^{36}}}$ **16.** $\dfrac{21\left(x^{-3/2}\right)\left(\sqrt{y}\right)\left(z^{5/2}\right)}{7^{-1}\sqrt{x}\left(y^{-1/2}\right)z}$

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Practice B

In Exercises 1–6, use the properties of rational exponents to simplify the expression.

1. $\dfrac{2^{2/5}}{2}$

2. $\left(\dfrac{3^6}{12^6}\right)^{-1/6}$

3. $\left(11^{3/2} \bullet 11^{-5/2}\right)^{-1/3}$

4. $\left(9^{-3/5} \bullet 9^{1/5}\right)^{-1}$

5. $\dfrac{3^{3/4} \bullet 27^{3/4}}{9^{3/4}}$

6. $\dfrac{25^{5/9} \bullet 25^{7/9}}{5^{4/3}}$

In Exercises 7–12, use the properties of radicals to simplify the expression.

7. $\sqrt[3]{25} \bullet \sqrt[3]{625}$

8. $\sqrt[5]{6} \bullet \sqrt[5]{81}$

9. $\dfrac{\sqrt[4]{176}}{\sqrt[4]{11}}$

10. $\dfrac{\sqrt{7}}{\sqrt{700}}$

11. $\dfrac{\sqrt[3]{5} \bullet \sqrt[3]{50}}{\sqrt[3]{2}}$

12. $\dfrac{\sqrt[4]{4} \bullet \sqrt[4]{12}}{\sqrt[8]{3} \bullet \sqrt[8]{3}}$

In Exercises 13–18, write the expression in simplest form.

13. $\dfrac{\sqrt[3]{4}}{\sqrt[3]{9}}$

14. $\sqrt[3]{\dfrac{4}{25}}$

15. $\sqrt[4]{\dfrac{2401}{4}}$

16. $\dfrac{7}{5 - \sqrt{3}}$

17. $\dfrac{6}{\sqrt{2} + \sqrt{7}}$

18. $\dfrac{\sqrt{2}}{\sqrt{15} - \sqrt{3}}$

In Exercises 19–24, simplify the expression.

19. $10\left(25^{2/3}\right) - 6\left(25^{2/3}\right)$

20. $2\sqrt{54} - 11\sqrt{6}$

21. $13\sqrt[3]{3} - \sqrt[3]{375}$

22. $\sqrt[5]{486} + 10\sqrt[5]{2}$

23. $4\left(48^{1/4}\right) - 3\left(3^{1/4}\right)$

24. $\left(7^{1/3}\right) + 4\left(189^{1/3}\right)$

25. The volume of a right circular cylinder is $V = 9\pi r^2$, where r is the radius.

a. Use radicals to solve $V = 9\pi r^2$ for r. Simplify, if possible.

b. Substitute the expression for r from part (a) into the formula for the surface area of a right cylinder, $S = 18\pi r + \pi r^2$.

c. Use the answer to part (b) to find the surface area of a right cylinder when the volume is 108 cubic meters.