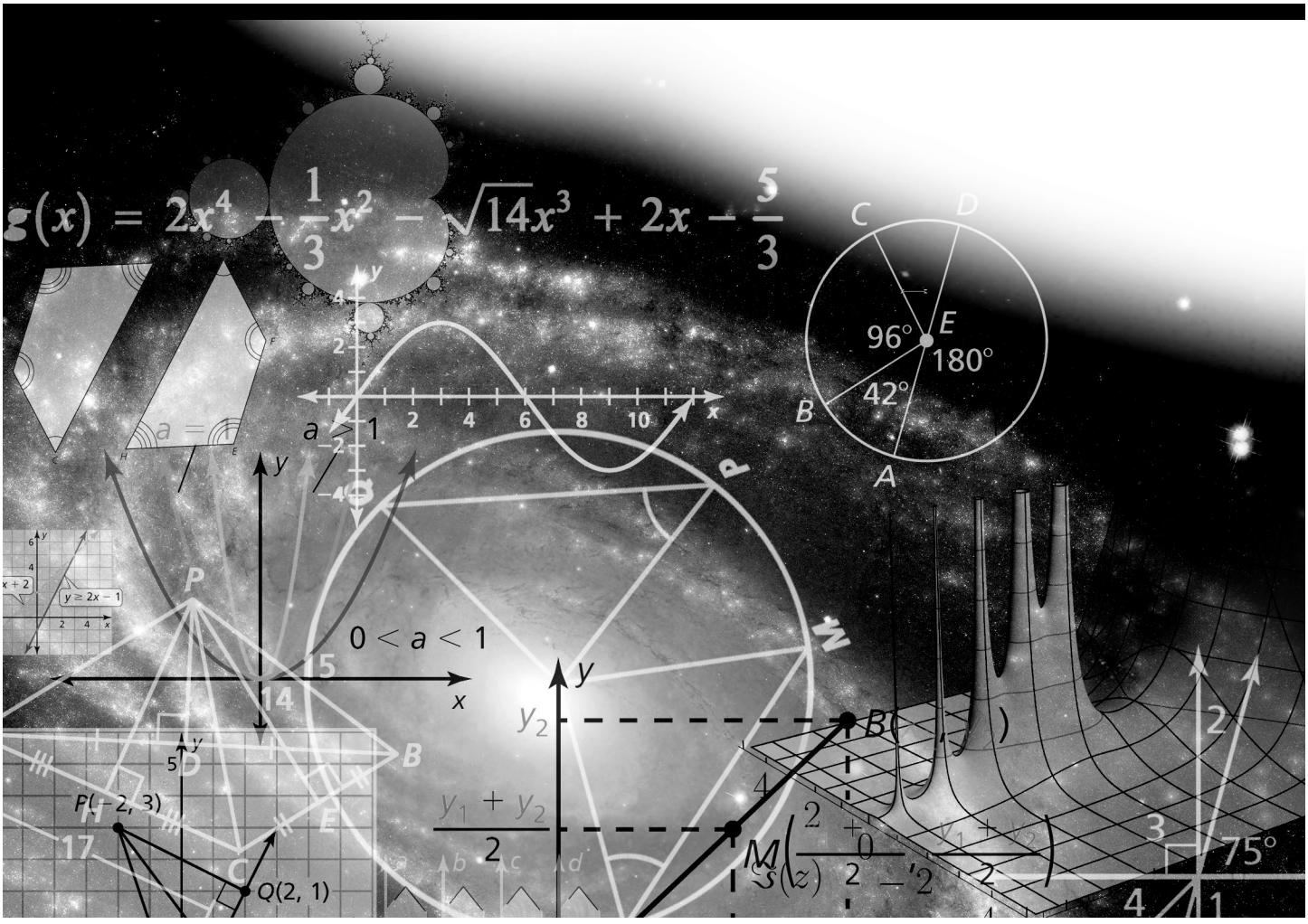
CHAPTER 3

Radical and Rational Functions



Name _____ Date _____

Chapter 3 Maintaining Mathematical Proficiency

Simplify the expression.

1. $c \bullet c^9$

2. $\frac{q^{12}}{q^4}$

3. $\frac{x^3}{x^4 \bullet x^5}$

4. $\frac{d^2}{d} \bullet 8d^5$

5. $\left(\frac{4x^3}{2y^4}\right)^2$

6. $\left(\frac{m^8 \bullet m^3}{n \bullet m}\right)^3$

Solve the literal equation for *y*.

7. $x + y = 1$

8. $-3y + \frac{1}{2}x = -6$

9. $24x + 5y = 74$

10. $6xy + 3y = -72$

11. $10x - 5xy = 100$

12. $-\frac{1}{4}x + 8xy = 16$

13. Is $\left(\frac{x + 3x}{y + 2y}\right)^2 = \left(\frac{x^2 + 3^2x^2}{y^2 + 2^2y^2}\right)$ or is $\left(\frac{x + 3x}{y + 2y}\right)^2 = \left(\frac{4^2x^2}{3^2y^2}\right)$? Explain your reasoning.

Name_____ Date _____

3.1 *n*th Roots and Rational Exponents

For use with Exploration 3.1

Essential Question How can you use a rational exponent to represent a power involving a radical?

1 EXPLORATION: Exploring the Definition of a Rational Exponent

Work with a partner. Use a calculator to show that each statement is true.

a. $\sqrt{9} = 9^{1/2}$ **b.** $\sqrt{2} = 2^{1/2}$ **c.** $\sqrt[3]{8} = 8^{1/3}$

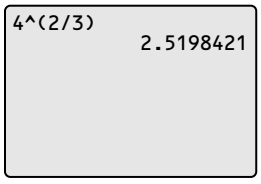
d. $\sqrt[3]{3} = 3^{1/3}$ **e.** $\sqrt[4]{16} = 16^{1/4}$ **f.** $\sqrt[4]{12} = 12^{1/4}$

2 EXPLORATION: Writing Expressions in Rational Exponent Form

Work with a partner. Use the definition of a rational exponent and the properties of exponents to write each expression as a base with a single rational exponent. Then use a calculator to evaluate each expression. Round your answer to two decimal places.

Sample

$$\left(\sqrt[3]{4}\right)^2 = \left(4^{1/3}\right)^2$$
$$= 4^{2/3}$$
$$\approx 2.52$$



a. $\left(\sqrt{5}\right)^3$ **b.** $\left(\sqrt[4]{4}\right)^2$ **c.** $\left(\sqrt[3]{9}\right)^2$

d. $\left(\sqrt[5]{10}\right)^4$ **e.** $\left(\sqrt{15}\right)^3$ **f.** $\left(\sqrt[3]{27}\right)^4$

3 EXPLORATION: Writing Expressions in Radical Form

Work with a partner. Use the properties of exponents and the definition of a rational exponent to write each expression as a radical raised to an exponent. Then use a calculator to evaluate each expression. Round your answer to two decimal places.

Sample $5^{2/3} = \left(5^{1/3}\right)^2 = \left(\sqrt[3]{5}\right)^2 \approx 2.92$

a. $8^{2/3}$ **b.** $6^{5/2}$ **c.** $12^{3/4}$

d. $10^{3/2}$ **e.** $16^{3/2}$ **f.** $20^{6/5}$

Communicate Your Answer

4. How can you use a rational exponent to represent a power involving a radical?

5. Evaluate each expression *without* using a calculator. Explain your reasoning.

a. $4^{3/2}$ **b.** $32^{4/5}$ **c.** $625^{3/4}$

d. $49^{3/2}$ **e.** $125^{4/3}$ **f.** $100^{6/3}$

Name_____ Date _____

3.1 Practice
For use after Lesson 3.1

Core Concepts

Real *n*th roots of *a*

Let n be an integer $(n > 1)$ and let a be a real number.

***n* is an even integer.**	***n* is an odd integer.**
$\boldsymbol{a < 0}$ No real nth roots	$\boldsymbol{a < 0}$ One real nth root: $\sqrt[n]{a} = a^{1/n}$
$\boldsymbol{a = 0}$ One real nth root: $\sqrt[n]{0} = 0$	$\boldsymbol{a = 0}$ One real nth root: $\sqrt[n]{0} = 0$
$\boldsymbol{a > 0}$ Two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$	$\boldsymbol{a > 0}$ One real nth root: $\sqrt[n]{a} = a^{1/n}$

Notes:

Rational Exponents

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

$$a^{m/n} = \left(a^{1/n}\right)^m = \left(\sqrt[n]{a}\right)^m$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{\left(a^{1/n}\right)^m} = \frac{1}{\left(\sqrt[n]{a}\right)^m}, a \neq 0$$

Notes:

Name _____ Date _____

3.1 Practice (continued)

Worked-Out Examples

Example #1

Find the indicated real nth root(s) of a.

$n = 4, a = 256$

Because $n = 4$ is even and $a = 256 > 0$, 256 has two real fourth roots. Because $4^4 = 256$ and $(-4)^4 = 256$, you can write $\pm\sqrt[4]{256} = \pm 4$ or $\pm 256^{1/4} = \pm 4$.

Example #2

Evaluate the expression using a calculator. Round your answer to two decimal places when appropriate.

$\sqrt[5]{32{,}768} = 8$

Practice A

In Exercises 1–3, find the indicated real *n*th root(s) of *a*.

1. $n = 3, a = -125$ **2.** $n = 2, a = -400$ **3.** $n = 6, a = 64$

In Exercises 4–11, evaluate the expression without using a calculator.

4. $64^{1/2}$ **5.** $(-27)^{1/3}$ **6.** $32^{7/5}$ **7.** $49^{-3/2}$

8. $(-32)^{3/5}$ **9.** $1000^{-2/3}$ **10.** $81^{3/4}$ **11.** $625^{1/4}$

Name_____ Date _____

3.1 Practice (continued)

In Exercises 12–15, match the equivalent expressions. Explain your reasoning.

12. $\left(\sqrt{a}\right)^3$

13. $-\sqrt[3]{a}$

14. $\left(\sqrt[3]{a}\right)^2$

15. $\dfrac{1}{\sqrt[3]{a}}$

A. $a^{-1/3}$

B. $a^{2/3}$

C. $a^{3/2}$

D. $-a^{1/3}$

In Exercises 16–19, find the real solution(s) of the equation. Round your answer to two decimal places when appropriate.

16. $6x^3 = -6$

17. $2(x + 5)^4 = 128$

18. $x^5 - 32 = -64$

19. $-\frac{1}{10}x^3 + 100 = 0$

20. The volume of a cube is 1728 cubic inches. What are the dimensions of the cube?

Name_____ Date _____

Practice B

In Exercises 1–3, find the indicated real *n*th root(s) of *a*.

1. $n = 3, a = 343$ **2.** $n = 6, a = -64$ **3.** $n = 5, a = -243$

In Exercises 4–9, evaluate the expression without using a calculator.

4. $36^{3/2}$ **5.** $16^{3/4}$ **6.** $(-32)^{2/5}$

7. $(-125)^{5/3}$ **8.** $256^{-5/4}$ **9.** $27^{-4/3}$

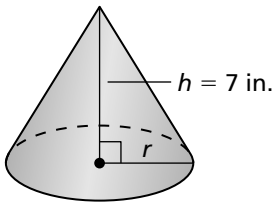
In Exercises 10–15, evaluate the expression using a calculator. Round your answer to two decimal places when appropriate.

10. $28^{-1/5}$ **11.** $150^{2/5}$ **12.** $40{,}351^{6/7}$

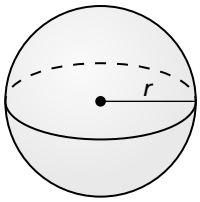
13. $750^{-2/5}$ **14.** $\left(\sqrt[5]{223}\right)^3$ **15.** $\left(\sqrt[7]{-34}\right)^5$

In Exercises 16 and 17, find the radius of the figure with the given volume.

16. $V = 425$ in.^{3}



17. $V = 1458$ m^3



In Exercises 18–23, find the real solution(s) of the equation. Round your answer to two decimal places when appropriate.

18. $6x^4 = 60$ **19.** $x^5 = -233$ **20.** $x^4 + 19 = 100$

21. $x^3 + 17 = 57$ **22.** $\frac{1}{5}x^4 = 125$ **23.** $\frac{1}{7}x^3 = -49$

24. Kepler's third law states that the relationship between the mean distance d (in astronomical units) of a planet from the Sun and the time t (in years) it takes the planet to orbit the Sun can be given by $d^3 = t^2$.

a. It takes Venus 0.616 year to orbit the Sun. Find the mean distance of Venus from the Sun (in astronomical units).

b. The mean distance of Jupiter from the Sun is 5.24 astronomical units. How many years does it take Jupiter to orbit the Sun?