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2.8 Quadratic Inequalities

For use with Exploration 2.8

Essential Question How can you solve a quadratic inequality?

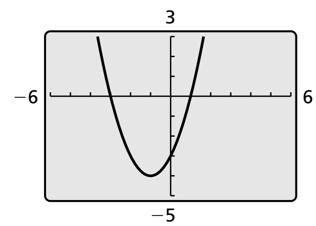
1 EXPLORATION: Solving a Quadratic Inequality

Work with a partner. The graphing calculator screen shows the graph of

$f(x) = x^2 + 2x - 3.$

Explain how you can use the graph to solve the inequality

$x^2 + 2x - 3 \leq 0.$



Then solve the inequality.

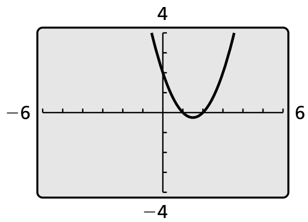
2 EXPLORATION: Solving Quadratic Inequalities

Work with a partner. Match each inequality with the graph of its related quadratic function on the next page. Then use the graph to solve the inequality.

a. $x^2 - 3x + 2 > 0$ **b.** $x^2 - 4x + 3 \leq 0$ **c.** $x^2 - 2x - 3 < 0$

d. $x^2 + x - 2 \geq 0$ **e.** $x^2 - x - 2 < 0$ **f.** $x^2 - 4 > 0$

A.



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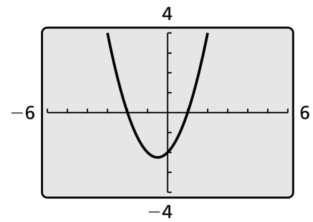

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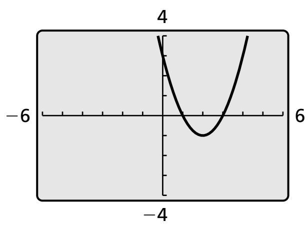
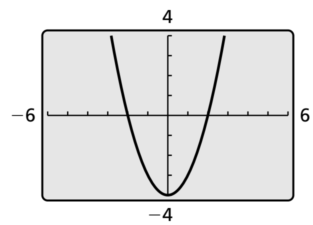
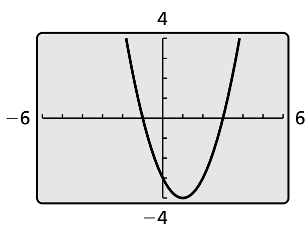


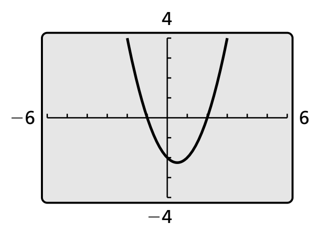
D.



E.



F.



Communicate Your Answer

3. How can you solve a quadratic inequality?

4. Explain how you can use the graph in Exploration 1 to solve each inequality. Then solve each inequality.

a. $x^2 + 2x - 3 > 0$ **b.** $x^2 + 2x - 3 < 0$ **c.** $x^2 + 2x - 3 \geq 0$

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2.8 Practice
For use after Lesson 2.8

Core Concepts

Graphing a Quadratic Inequality in Two Variables

To graph a quadratic inequality in one of the following forms,

$y < ax^2 + bx + c$ $\quad$ $y > ax^2 + bx + c$

$y \leq ax^2 + bx + c$ $\quad$ $y \geq ax^2 + bx + c$,

follow these steps.

Step 1 Graph the parabola with the equation $y = ax^2 + bx + c$. Make the parabola *dashed* for inequalities with < or > and *solid* for inequalities with ≤ or ≥.

Step 2 Test a point (x, y) inside the parabola to determine whether the point is a solution of the inequality.

Step 3 Shade the region inside the parabola if the point from Step 2 is a solution. Shade the region outside the parabola if it is not a solution.

Notes:

Worked-Out Examples

Example #1

Graph the inequality.

$y > 2(x + 3)^2 - 1$

Step 1 Graph $y = 2(x + 3)^2 - 1$. Because the inequality symbol is $>$, make the parabola dashed.

Step 2 Test a point inside the parabola, such as $(-3, 1)$.

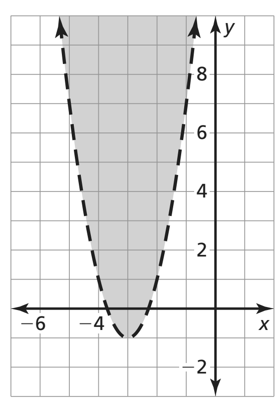
$y > 2(x + 3)^2 - 1$

$1 \overset{?}{>} 2(-3 + 3)^2 - 1$

$1 > -1$

So, $(-3, 1)$ is a solution of the inequality.

Step 3 Shade the region inside the parabola.



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2.8 Practice (continued)

Example #2

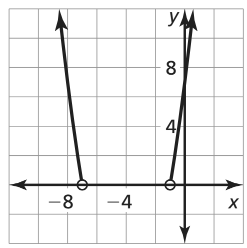
Solve the inequality by graphing.

$x^2 + 8x > -7$

The solution consists of the x-values for which the graph of $y = x^2 + 8x + 7$ lies above the x-axis. Find the x-intercepts of the graph by letting $y = 0$ and use factoring to solve $0 = x^2 + 8x + 7$ for x.

$$x^2 + 8x + 7 = 0$$
$$(x + 7)(x + 1) = 0$$
$$x + 7 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = -7 \quad \text{or} \quad x = -1$$

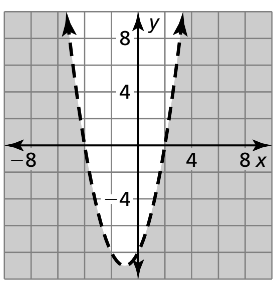
The solutions are $x = -7$ and $x = -1$. Sketch a parabola that opens up and has -7 and -1 as x-intercepts. The graph lies above the x-axis to the left of $x = -7$ and to the right of $x = -1$. The solution of the inequality is $x < -7$ or $x > -1$.



Practice A

In Exercises 1–4, match the graph with its inequality. Explain your reasoning.

1.



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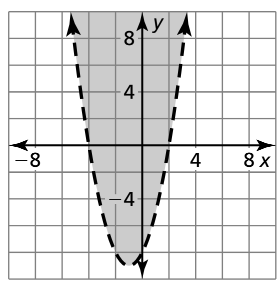
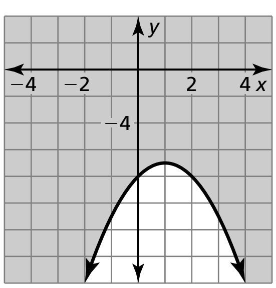
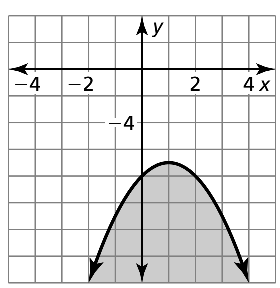


3.



4.



A. $y < x^2 + 2x - 8$

B. $y \leq -x^2 + 2x - 8$

C. $y > x^2 + 2x - 8$

D. $y \geq -x^2 + 2x - 8$

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2.8 Practice (continued)

In Exercises 5–8, graph the inequality.

5. $y < x^2 + 2$ **6.** $y \le -5x^2$ **7.** $y \ge -(x+4)^2 - 1$ **8.** $y < 4x^2 + 4x + 1$

9. Accident investigators use the formula $d = 0.01875v^2$, where d is the braking distance of a car (in feet) and v is the speed of the car (in miles per hour) to determine how fast a car is going at the time of an accident. For what speeds v would a car leave a tire mark on the road of over 1 foot?

In Exercises 10–12, graph the system of quadratic inequalities.

10. $y \le -x^2$
$y > -3x^2 + 3$

11. $y \ge x^2 + 5x$
$y \ge (x+2)^2 - 1$

12. $y > x^2 - 7x - 8$
$y < -x^2 + 6x + 5$

In Exercises 13–15, solve the inequality algebraically.

13. $16x^2 > 100$ **14.** $x^2 \le 15x - 34$ **15.** $-\frac{1}{5}x^2 + 10x \ge -25$

16. The profit for a hot dog company is given by the equation $y = -0.02x^2 + 140x - 2500$, where x is the number of hot dogs produced and y is the profit (in dollars). How many hot dogs must be produced so that the company will generate a profit of at least \$150,000?

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Practice B

In Exercises 1–4, graph the inequality.

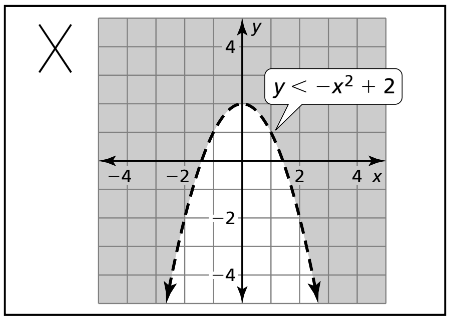
1. $y \le x^2 + 3$

2. $y > x^2 + 2x - 3$

3. $y < -(x + 1)^2 + 2$

4. $y \ge -x^2 + 4x$

5. Describe and correct the error in graphing $y < -x^2 + 2$.



In Exercises 6 and 7, graph the system of quadratic inequalities.

6. $y \le -x^2 + 3$
$y \ge 2x^2 - 3x + 1$

7. $y > x^2 - x + 4$
$y < x^2 + 2x - 4$

In Exercises 8–11, solve the inequality algebraically.

8. $2x^2 - 6 > -11x$

9. $2x^2 - 5x + 3 \le 1$

10. $\frac{1}{2}x^2 + 3x \ge 2$

11. $\frac{1}{3}x^2 - 2x < 9$

In Exercises 12–15, solve the inequality by graphing.

12. $2x^2 - 6 > -3x$

13. $4x^2 + 3x - 5 \le 1$

14. $\frac{1}{2}x^2 + x \le 2$

15. $\frac{2}{3}x^2 + 2x > 4$

16. An object is dropped from a building. The height h (in feet) of the object after t seconds can be modeled by $h(t) = -16t^2 - 28t + 25$.

a. At what height was the object initially dropped? Explain.

b. Write an inequality that you can use to find the t-values for which the object was in the air.

c. Based on your results from parts (a) and (b), use a graphing calculator to determine the time intervals in which the object was in the air.