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2.6 Solving Quadratic Equations with Complex Solutions

For use with Exploration 2.6

Essential Question How can you determine whether a quadratic equation has real solutions or imaginary solutions?

1 EXPLORATION: Using Graphs to Solve Quadratic Equations

Work with a partner. Use the discriminant of $f(x) = 0$ and the sign of the leading coefficient of $f(x)$ to match each quadratic function with its graph. Explain your reasoning. Then find the real solution(s) (if any) of each quadratic equation $f(x) = 0$.

a. $f(x) = x^2 - 2x$

b. $f(x) = x^2 - 2x + 1$

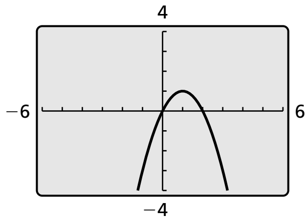
c. $f(x) = x^2 - 2x + 2$

d. $f(x) = -x^2 + 2x$

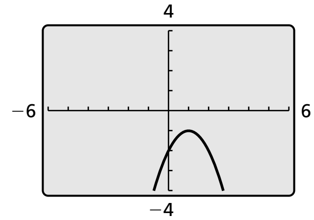
e. $f(x) = -x^2 + 2x - 1$

f. $f(x) = -x^2 + 2x - 2$

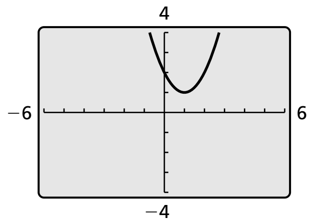
A.



B.



C.



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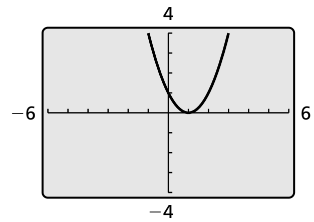

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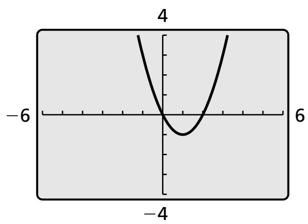

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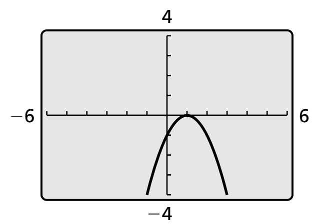

2 EXPLORATION: Finding Imaginary Solutions

Work with a partner. What do you know about the discriminants of quadratic equations that have no real solutions? Use the Quadratic Formula and what you learned about the imaginary unit *i* to find the *imaginary* solutions of each equation in Exploration 1 that has no real solutions. Use substitution to check your answer.

Communicate Your Answer

3. How can you determine whether a quadratic equation has real solutions or imaginary solutions?

4. Describe the number and type of solutions of $x^2 + 2x + 3 = 0$. How do you know? What are the solutions?

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2.6 Practice

For use after Lesson 2.6

Notes:

Worked-Out Examples

Example #1

Solve the equation using any method. Explain your choice of method.

$6x^2 - 2x + 1 = 0$

Sample answer: The equation is not factorable, and completing the square would result in fractions. So, solve using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(6)(1)}}{2(6)}$$

$$x = \frac{2 \pm \sqrt{-20}}{12}$$

$$x = \frac{2 \pm 2i\sqrt{5}}{12}$$

$$x = \frac{1 \pm i\sqrt{5}}{6}$$

So, the solutions are $x = \frac{1 + i\sqrt{5}}{6}$ and $x = \frac{1 - i\sqrt{5}}{6}$.

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Example #2

Solve the equation using any method. Explain your choice of method.

$3x^2 + 87 = 30x$

Sample answer: The equation can be factored by a GCF of 3. Then, divide each side by 3, and as a result, the coefficient of the x^2-term is 1, and the coefficient of the x-term is an even number. So, solve by completing the square.

$$\begin{aligned}
3x^2 + 87 - 30x &= 30x - 30x \\
3x^2 - 30x + 87 &= 0 \\
3(x^2 - 10x + 29) &= 0 \\
x^2 - 10x + 29 &= 0 \\
x^2 - 10x + 29 - 29 &= 0 - 29 \\
x^2 - 10x &= -29 \\
x^2 - 10x + 25 &= -29 + 25 \\
(x - 5)^2 &= -4 \\
x - 5 &= \pm\sqrt{-4} \\
x - 5 &= \pm 2i \\
x - 5 + 5 &= \pm 2i + 5 \\
x &= 5 \pm 2i
\end{aligned}$$

So, the solutions are $x = 5 + 2i$ and $x = 5 - 2i$.

Practice A

In Exercises 1–3, solve the equation using the Quadratic Formula.

1. $x^2 - 7x - 18 = 0$
2. $w^2 = 4w - 1$
3. $-7z = -4z^2 - 3$

In Exercises 4–7, determine whether you would use factoring, square roots, or completing the square to solve the equation. Explain your reasoning. Then solve the equation.

4. $x^2 + 7x = 0$
5. $(x - 1)^2 = 35$
6. $x^2 - 255 = 0$
7. $4x^2 + 8x + 12 = 0$

8. A baseball player hits a foul ball straight up in the air from a height of 4 feet off the ground with an initial velocity of 85 feet per second.

a. Write a quadratic function that represents the height h of the ball t seconds after it hits the bat.

b. When is the ball 110 feet off the ground? Explain your reasoning.

c. The catcher catches the ball 6 feet from the ground. How long is the ball in the air?

9. A golfer hits a golf ball on the fairway with an initial velocity of 80 feet per second. The height h (in feet) of the golf ball t seconds after it is hit can be modeled by the function $h(t) = -16t^2 + 80t + 0.1$.

a. Find the maximum height of the golf ball.

b. How long does the ball take to hit the ground?

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Practice B

In Exercises 1–8, solve the equation using any method. Explain your choice of method.

1. $x^2 + 16 = -28$

2. $\frac{1}{3}x^2 = -15$

3. $k^2 - 16k + 64 = -8$

4. $t^2 - 30t + 225 = -24$

5. $x^2 + 5x + 20 = 0$

6. $4x^2 - 3x - 5 = 0$

7. $3x^2 - 6x = -25$

8. $-3t^2 = -8t + 6$

9. Write a quadratic equation in the form $x^2 + bx + c = 0$ that has the solutions $x = -5 \pm i$.

In Exercises 10–15, find the zeros of the function.

10. $f(x) = -x^2 - 48$

11. $g(x) = -\frac{1}{4}x^2 - 13$

12. $f(x) = 7x^2 + 3x + 6$

13. $f(x) = x^2 + 100$

14. $p(x) = x^2 + x + 2$

15. $w(x) = -3x^2 + 3x - 4$

In Exercises 16 and 17, find a possible pair of integer values for *a* and *c* so that the quadratic equation has the given solution(s). Then write the equation.

16. $ax^2 - 3x + c = 0$; two real solutions

17. $ax^2 + 10x + c = 0$; two imaginary solutions

18. Your friend says that the situation representing the height y (in feet) of a basketball t seconds after it is thrown can be modeled by the function $y = -16t^2 + 12t + 3$. Is it possible for the basketball to reach the height of 6 feet? Explain.

In Exercises 19 and 20, use the Quadratic Formula to write a quadratic equation that has the given solutions.

19. $x = \dfrac{10 \pm \sqrt{-68}}{14}$

20. $x = \dfrac{-3 \pm 5i}{8}$

21. Suppose a quadratic equation has the form $x^2 + x + c = 0$. Show that the constant c must be greater than $\frac{1}{4}$ in order for the equation to have two imaginary solutions.