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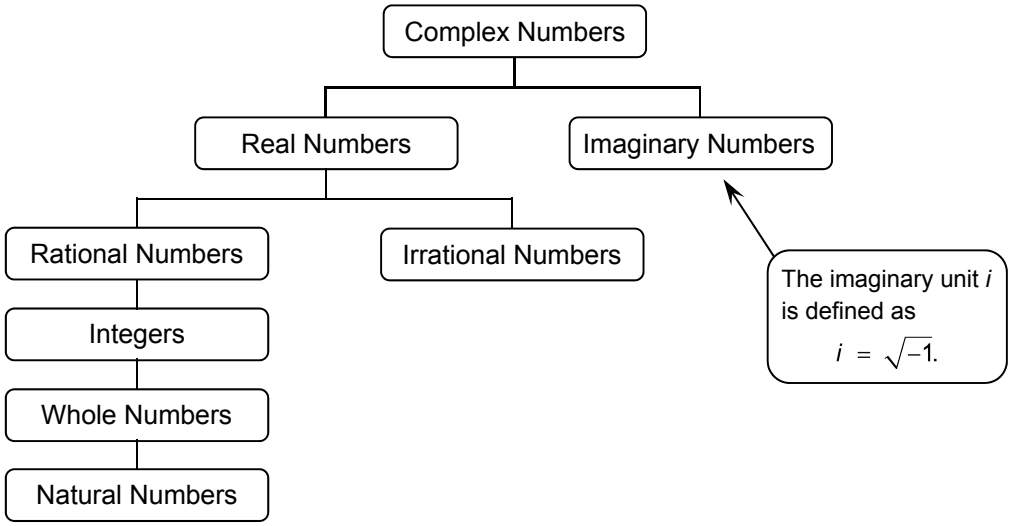
2.5 Complex Numbers

For use with Exploration 2.5

Essential Question What are the subsets of the set of complex numbers?



1 EXPLORATION: Classifying Numbers

Work with a partner. Determine which subsets of the set of complex numbers contain each number.

a. $\sqrt{9}$ **b.** $\sqrt{0}$ **c.** $-\sqrt{4}$

d. $\sqrt{\frac{4}{9}}$ **e.** $\sqrt{2}$ **f.** $\sqrt{-1}$

2 EXPLORATION: Simplifying i^2

Work with a partner. Justify each step in the simplification of i^2.

Algebraic Step	Justification
$i^2 = \left(\sqrt{-1}\right)^2$	_____
$= -1$	_____

Communicate Your Answer

3. What are the subsets of the set of complex numbers? Give an example of a number in each subset.

4. Is it possible for a number to be both whole and natural? natural and rational? rational and irrational? real and imaginary? Explain your reasoning.

5. Your friend claims that the conclusion in Exploration 2 is incorrect because $i^2 = i \bullet i = \sqrt{-1} \bullet \sqrt{-1} = \sqrt{-1(-1)} = \sqrt{1} = 1$. Is your friend correct? Explain.

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2.5 Practice
For use after Lesson 2.5

Core Concepts

The Square Root of a Negative Number

Property	Example
1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.	$\sqrt{-3} = i\sqrt{3}$
2. By the first property, it follows that $\left(i\sqrt{r}\right)^2 = -r$.	$\left(i\sqrt{3}\right)^2 = i^2 \bullet 3 = -3$

Notes:

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

Notes:

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2.5 Practice (continued)

Worked-Out Examples

Example #1

Find the values of x and y that satisfy the equation.

$-10x + 12i = 20 + 3yi$

Set the real parts equal to each other and the imaginary parts equal to each other.

$-10x = 20 \qquad 12 = 3y$

$x = -2 \qquad y = 4$

So, $x = -2$ and $y = 4$.

Example #2

Add or subtract. Write the answer in standard form.

$16 - (2 - 3i) - i = (16 - 2) + (3 - 1)i$

$= 14 + 2i$

Practice A

In Exercises 1–6, find the square root of the number.

1. $\sqrt{-49}$ **2.** $\sqrt{-4}$ **3.** $\sqrt{-45}$

4. $-2\sqrt{-100}$ **5.** $6\sqrt{-121}$ **6.** $5\sqrt{-75}$

In Exercises 7 and 8, find the values of *x* and *y* that satisfy the equation.

7. $-10x + i = 30 - yi$ **8.** $44 - \frac{1}{2}yi = -\frac{1}{4}x - 7i$

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2.5 Practice (continued)

In Exercises 9–14, simplify the expression. Then classify the result as a *real number* or *imaginary number*. If the result is an *imaginary number*, specify if it is a *pure imaginary number*.

9. $(-8 + 3i) + (-1 - 2i)$

10. $(36 - 3i) - (12 + 24i)$

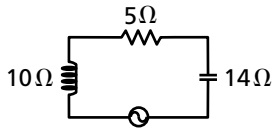
11. $(16 + i) + (-16 - 8i)$

12. $(-5 - 5i) - (-6 - 6i)$

13. $(-1 + 9i)(15 - i)$

14. $(6 - 7i)(-5 + 8i)$

15. Find the impedance of the series circuit.



In Exercises 16–18, multiply the complex number by this complex conjugate.

16. $8 + i$

17. $3 - 2i$

18. $-7 - 5i$

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Practice B

In Exercises 1–3, find the square root of the number.

1. $3\sqrt{-25}$ **2.** $2\sqrt{-40}$ **3.** $4\sqrt{-54}$

In Exercises 4–7, find the values of *x* and *y* that satisfy the equation.

4. $2x - 3yi = 14 + 12i$ **5.** $\frac{1}{3}x - 6i = 8 - 3yi$

6. $22 + \frac{1}{5}yi = 2x - 2$ **7.** $-1 + 10i = -x + 3yi$

In Exercises 8–11, add or subtract. Write the answer in standard form.

8. $(9 + 6i) - (15 - 7i)$ **9.** $13 - (5 + i) + 7i$

10. $14 - (17 - 7i) + 8i$ **11.** $-4 + (9 - 2i) + 3i$

12. The additive inverse of a complex number z is a complex number z_a such that $z + z_a = 0$. Find the additive inverse of each complex number.

a. $z = 2 + 3i$ **b.** $z = 4 - 4i$ **c.** $z = -5 + 2i$

In Exercises 13–16, multiply. Write the answer in standard form.

13. $(4 + 7i)(5 + 2i)$ **14.** $(5 - 3i)(5 + 3i)$

15. $(10 - 7i)(10 + 7i)$ **16.** $(6 - 4i)^2$

17. Justify each step in performing the operation.

$(6 - 2i)(8 - 3i)$

$48 - 18i - 16i + 6i^2$	
$48 - 34i + 6i^2$	
$48 - 34i + 6(-1)$	
$42 - 34i$	

In Exercises 18–20, multiply the complex number by its complex conjugate.

18. $4 + 5i$ **19.** $6 - 4i$ **20.** $-2 - 5i$

21. Write the complex conjugate of $1 + \sqrt{-18}$. Then find the product of the complex conjugates.