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2.3 Solving Quadratic Equations by Completing the Square

For use with Exploration 2.3

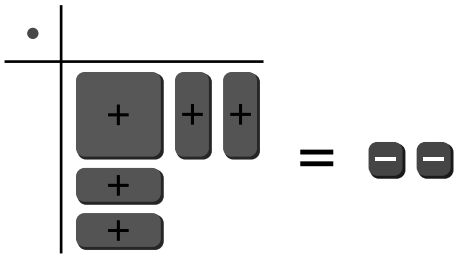
Essential Question How can you use "completing the square" to solve a quadratic equation?

1 EXPLORATION: Solving by Completing the Square

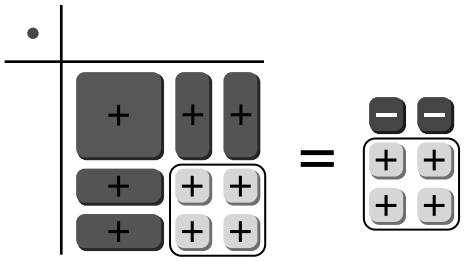
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

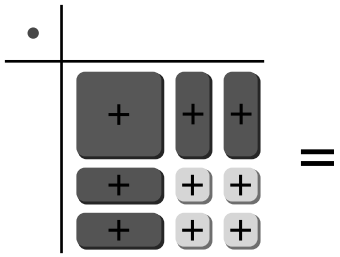
a. Write the equation modeled by the algebra tiles. This is the equation to be solved.



b. Four algebra tiles are added to the left side to "complete the square." Why are four algebra tiles also added to the right side?



c. Use algebra tiles to label the dimensions of the square on the left side and simplify on the right side.



d. Write the equation modeled by the algebra tiles so that the left side is the square of a binomial. Solve the equation using square roots.

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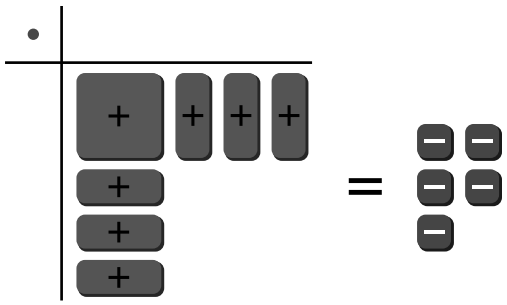
2.3 Solving Quadratic Equations by Completing the Square (continued)

2 EXPLORATION: Solving by Completing the Square

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Write the equation modeled by the algebra tiles.



b. Use algebra tiles to "complete the square."

c. Write the solutions of the equation.

d. Check each solution in the original equation.

Communicate Your Answer

3. How can you use "completing the square" to solve a quadratic equation?

4. Solve each quadratic equation by completing the square.

a. $x^2 - 2x = 1$ **b.** $x^2 - 4x = -1$ **c.** $x^2 + 4x = -3$

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2.3 Practice
For use after Lesson 2.3

Core Concepts

Completing the Square

Words To complete the square for an expression of the form $x^2 + bx$, follow these steps.

Step 1 Find one-half of b, the coefficient of x.

Step 2 Square the result from Step 1.

Step 3 Add the result from Step 2 to $x^2 + bx$.

Factor the resulting expression as the square of a binomial.

Algebra $x^2 + \boldsymbol{b}x + \left(\frac{\boldsymbol{b}}{2}\right)^2 = \left(x + \frac{\boldsymbol{b}}{2}\right)^2$

Notes:

Worked-Out Examples

Example #1

Complete the square for the expression. Then factor the trinomial.

$x^2 + 22x$

Step 1 $\frac{b}{2} = \frac{22}{2} = 11$

Step 2 $11^2 = 121$

Step 3 $x^2 + 22x + 121$

So, $x^2 + 22x + 121 = (x + 11)^2$.

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2.3 Practice (continued)

Example #2

Solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary

$$2x^2 - 14x + 10 = 26$$
$$\underline{-10} \quad \underline{-10}$$
$$2x^2 - 14x = 16$$
$$\frac{2x^2 - 14x}{2} = \frac{16}{2}$$
$$x^2 - 7x = 8$$

$$x^2 - 7x + \left(-\frac{7}{2}\right)^2 = 8 + \left(-\frac{7}{2}\right)^2$$
$$\left(x - \frac{7}{2}\right)^2 = 8 + \frac{49}{4}$$
$$\left(x - \frac{7}{2}\right)^2 = \frac{81}{4}$$
$$\sqrt{\left(x - \frac{7}{2}\right)^2} = \sqrt{\frac{81}{4}}$$
$$x - \frac{7}{2} = \pm\frac{9}{2}$$
$$\underline{+\frac{7}{2}} \quad \underline{+\frac{7}{2}}$$
$$x = \frac{7}{2} \pm \frac{9}{2}, \text{ or } \frac{7 \pm 9}{2}$$

The solutions are $x = \frac{7 + 9}{2} = \frac{16}{2} = 8$ and $x = \frac{7 - 9}{2} = \frac{-2}{2} = -1$.

Practice A

In Exercises 1–6, complete the square for the expression. Then factor the trinomial.

1. $x^2 + 12x$

2. $x^2 - 14x$

3. $x^2 + 4x$

4. $x^2 + 18x$

5. $x^2 - 7x$

6. $x^2 + 11x$

In Exercises 7–18, solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

7. $x^2 - 8x = -15$

8. $x^2 + 2x = 3$

9. $x^2 + 7x = 30$

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2.3 Practice (continued)

10. $x^2 - 26x = -9$

11. $x^2 - 12x = 10$

12. $x^2 - 15x = 18$

13. $x^2 - 12x + 9 = 0$

14. $x^2 + 14x - 10 = 0$

15. $x^2 + 2x - 99 = 0$

16. $10x^2 - 13x - 9 = 0$

17. $3x^2 + 6x - 1 = 0$

18. $12x^2 - 8x - 2 = 0$

In Exercises 19–24, determine whether the quadratic function has a maximum or minimum value. Then find the value.

19. $y = -x^2 + 4x + 3$

20. $y = x^2 + 6x + 10$

21. $y = -x^2 + 8x - 2$

22. $y = x^2 - 10x + 8$

23. $y = 3x^2 + 3x - 1$

24. $y = -4x^2 + 8x + 12$

25. A diver jumps off a diving board. The function $h = -16x^2 + 6x + 5$ represents the height (in feet) of the diver after x seconds. What is the maximum height above the water of the diver? How many seconds did it take for the diver to reach the maximum height? Round your answers to the nearest hundredth.

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Practice B

In Exercises 1–3, find the value of *c* that completes the square.

1. $x^2 - 16x + c$ **2.** $x^2 - x + c$ **3.** $x^2 + 7x + c$

In Exercises 4–6, complete the square for the expression. Then factor the trinomial.

4. $x^2 - 14x$ **5.** $x^2 + 30x$ **6.** $x^2 - 9x$

In Exercises 7–9, solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

7. $x^2 + 10x = 16$ **8.** $x^2 - 3x = 7$ **9.** $x^2 + 15x = 12$

10. A wading pool is 1 foot deep and has a volume of 108 cubic feet. The width is 12 feet less than the length.

a. Write an equation that represents the volume of the wading pool.

b. Find the dimensions of the wading pool by completing the square.

In Exercises 11–16, solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

11. $x^2 - 10x + 17 = 0$ **12.** $x^2 + 22x + 25 = 0$

13. $3x^2 - 15x + 27 = 0$ **14.** $2x^2 + 40x + 32 = 0$

15. $-3x^2 - 12x - 10 = -37$ **16.** $5x^2 - 15x - 10 = 20$

17. Find all values of b for which $x^2 + bx + 49$ is a perfect square.

In Exercises 18–21, determine whether the quadratic function has a maximum or minimum value. Then find the value.

18. $y = x^2 - 6x + 4$ **19.** $y = 2x^2 + 16x - 7$

20. $y = -3x^2 - 15x - 21$ **21.** $y = 5x^2 - 20x + 25$

22. The product of two consecutive odd integers that are positive is 323.

a. Write an equation to find the integers.

b. Find the two integers.