

1.3

Solving Linear Systems

For use with Exploration 1.3

Essential Question How can you determine the number of solutions of a linear system?

A linear system is *consistent* when it has at least one solution. A linear system is *inconsistent* when it has no solution.

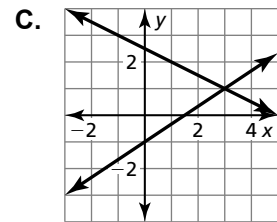
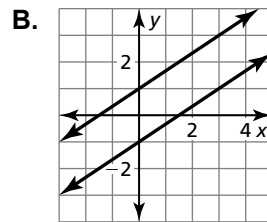
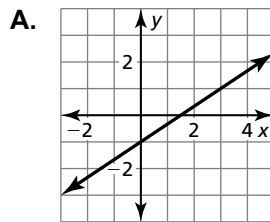
1 EXPLORATION: Recognizing Graphs of Linear Systems

Work with a partner. Match each linear system with its corresponding graph. Explain your reasoning. Then classify the system as *consistent* or *inconsistent*.

a. $2x - 3y = 3$
 $-4x + 6y = 6$

b. $2x - 3y = 3$
 $x + 2y = 5$

c. $2x - 3y = 3$
 $-4x + 6y = -6$



2 EXPLORATION: Solving Systems of Linear Equations

Work with a partner. Solve each linear system by substitution or elimination. Then use the graph of the system on the next page to check your solution.

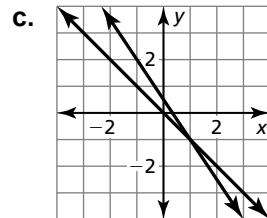
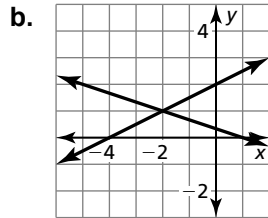
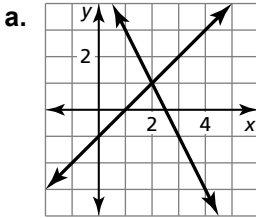
a. $2x + y = 5$
 $x - y = 1$

b. $x + 3y = 1$
 $-x + 2y = 4$

c. $x + y = 0$
 $3x + 2y = 1$

1.3 Solving Linear Systems (continued)

2 EXPLORATION: Solving Systems of Linear Equations (continued)



Communicate Your Answer

3. How can you determine the number of solutions of a linear system?

4. Suppose you were given a system of *three* linear equations in *three* variables. Explain how you would approach solving such a system.

5. Apply your strategy in Question 4 to solve the linear system.

$$\begin{array}{ll}
 x + y + z = 1 & \text{Equation 1} \\
 x - y - z = 3 & \text{Equation 2} \\
 -x - y + z = -1 & \text{Equation 3}
 \end{array}$$

1.3**Practice**

For use after Lesson 1.3

Core Concepts**Solving a Three-Variable System**

- Step 1** Rewrite the linear system in three variables as a linear system in two variables by using the substitution or elimination method.
- Step 2** Solve the new linear system for both of its variables.
- Step 3** Substitute the values found in Step 2 into one of the original equations and solve for the remaining variable.

When you obtain a false equation, such as $0 = 1$, in any of the steps, the system has no solution.

When you do not obtain a false equation, but obtain an identity such as $0 = 0$, the system has infinitely many solutions.

Notes:**Worked-Out Examples****Example #1**

Solve the system using the elimination method.

$$2x + y - z = 9$$

$$-x + 6y + 2z = -17$$

$$5x + 7y + z = 4$$

Step 1 Rewrite the system as a linear system in two variables.

$$2x + y - z = 9$$

$$5x + 7y + z = 4$$

$$7x + 8y = 13$$

$$4x + 2y - 2z = 18$$

$$-x + 6y + 2z = -17$$

$$3x + 8y = 1$$

Step 2 Solve the new linear system for both of its variables.

$$7x + 8y = 13$$

$$-3x - 8y = -1$$

$$4x = 12$$

$$x = 3$$

$$y = -1$$

1.3 Practice (continued)

Step 3 Substitute $x = 3$ and $y = -1$ into an original equation and solve for z .

$$2x + y - z = 9$$

$$2(3) + (-1) - z = 9$$

$$6 - 1 - z = 9$$

$$z = -4$$

The solution is $(3, -1, -4)$.

Example #2

Solve the system of linear equations using the substitution method.

$$x + y - z = 4$$

$$3x + 2y + 4z = 17$$

$$-x + 5y + z = 8$$

Step 1 Solve the first equation for x .

$$x = 4 - y + z$$

Step 2 Rewrite the system as a linear system in two variables by substituting for x from Step 1 into Equations 2 and 3.

$$3x + 2y + 4z = 17$$

$$-x + 5y + z = 8$$

$$3(4 - y + z) + 2y + 4z = 17$$

$$-(4 - y + z) + 5y + z = 8$$

$$12 - 3y + 3z + 2y + 4z = 17$$

$$-4 + y - z + 5y + z = 8$$

$$-y + 7z = 5$$

$$6y = 12$$

$$y = 2$$

Step 3 Solve the new linear system for the variables x and z .

$$y = 2$$

$$-y + 7z = 5$$

$$-2 + 7z = 5$$

$$7z = 7$$

$$z = 1$$

$$x = 3$$

The solution is $(3, 2, 1)$.

Practice A

In Exercises 1–3, solve the system using the elimination method.

1. $x + 2y - 3z = 11$

2. $x - y + 3z = 19$

3. $x + y - z = -9$

$$2x + y - 2z = 9$$

$$-2x + 2y - 6z = 9$$

$$2x - 3y + 2z = 13$$

$$4x + 3y + z = 16$$

$$3x + 5y + 2z = 3$$

$$3x - 5y - 6z = -15$$

1.3 Practice (continued)

In Exercises 4–6, solve the system using the substitution method.

4. $x + y + z = 4$

$x + y - z = 4$

$3x + 3y + z = 12$

5. $2x + 3y - z = 9$

$x - 3y + z = -6$

$3x + y - 4z = 31$

6. $x + 2y - 5z = -12$

$2x + 2y - 3z = -2$

$3x - 4y - z = 11$

7. You found \$6.60 on the ground at school, all in nickels, dimes, and quarters. You have twice as many quarters as dimes and 42 coins in all. How many of each type of coin do you have?

8. Find the values of a , b , and c so that the linear system below has $(3, -2, 1)$ as its only solution. Explain your reasoning.

$$3x + 2y - 7z = a$$

$$x + 3y + z = b$$

$$4x - 2y - z = c$$

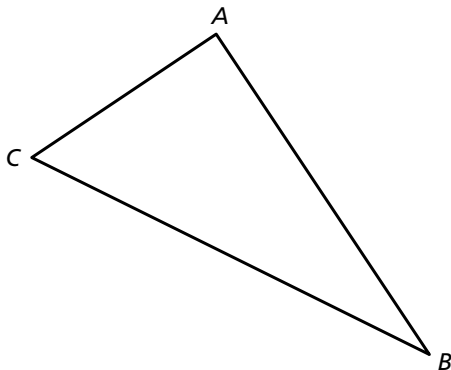
9. Does the system of linear equations have more than one solution? Justify your answer.

$$\frac{1}{2}x - \frac{3}{8}y + \frac{1}{8}z = -\frac{5}{4}$$

$$\frac{1}{2}x + \frac{1}{4}y + \frac{3}{4}z = 0$$

$$-x + 2y - 5z = 17$$

10. If $\angle A$ is three times as large as $\angle B$, and $\angle B$ is 30° smaller than $\angle C$, what are the measures of angles A , B , and C ?



Practice B

In Exercises 1 and 2, solve the system using the elimination method.

- | | |
|--|---|
| <p>1. $3x - y + z = -1$
 $3x + 2y - 5z = -16$
 $3x + 3y + 2z = 6$</p> | <p>2. $4x + 3y - 5z = -9$
 $6x + 6y - 3z = 6$
 $3x - 3y + 4z = 19$</p> |
|--|---|

3. Describe and correct the error in the first step of solving the system of linear equations.

$$\begin{aligned} 5x + 3y - z &= 15 \\ -x + 2y + 3z &= 10 \\ 3x - 4y + 3z &= 8 \end{aligned}$$

$\begin{array}{r} \times \\ -15x - 9y - 3z = 45 \\ \underline{3x - 4y + 3z = 8} \\ -12 - 13y = 53 \end{array}$
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In Exercises 4 and 5, solve the system using the elimination method.

- | | |
|---|---|
| <p>4. $x - y - z = 5$
 $4x - 4y - 4z = 15$
 $3x - y - 4z = -2$</p> | <p>5. $-x + y + z = 3$
 $x + y + 3z = 5$
 $3y + 6z = 12$</p> |
|---|---|

In Exercises 6 and 7, solve the system of linear equations using the substitution method.

- | | |
|---|--|
| <p>6. $2x - y = 6$
 $4x - 3y - 2z = 14$
 $-x + 2y - 3z = 12$</p> | <p>7. $6x + 3y - 9z = 10$
 $-2x - y + 3z = 3$
 $x - 2y - z = 1$</p> |
|---|--|

8. Your friend claims that she has a bag of 30 coins containing nickels, dimes, and quarters. The total value of the 30 coins is \$3. There are twice as many nickels as there are dimes. Is your friend correct? Explain your reasoning.

9. Each equation in this system represents a line.

$$\begin{aligned} x - 2y - 3 &= 0 \\ 2x + y + 1 &= 0 \\ 3x + 4y + 5 &= 0 \end{aligned}$$

- Solve the system of linear equations using either the elimination method or the substitution method.
- Do the lines intersect at a point? Explain.