Name ____ Date _________

11.3 Coordinate Proofs

For use with Exploration 11.3

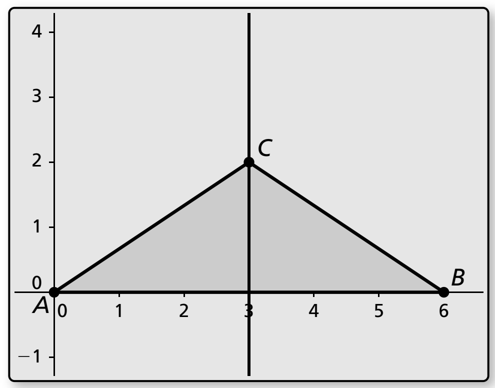
Essential Question How can you use a coordinate plane to write a proof?

1 EXPLORATION: Writing a Coordinate Proof

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to draw $\overline{AB}$ with endpoints $A(0, 0)$ and $B(6, 0)$.



Sample
Points
$A(0, 0)$
$B(6, 0)$
$C(3, y)$
Segments
$AB = 6$
Line
$x = 3$

b. Draw the vertical line $x = 3$.

c. Draw $\triangle ABC$ so that C lies on the line $x = 3$.

d. Use your drawing to prove that $\triangle ABC$ is an isosceles triangle.

2 EXPLORATION: Writing a Coordinate Proof

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

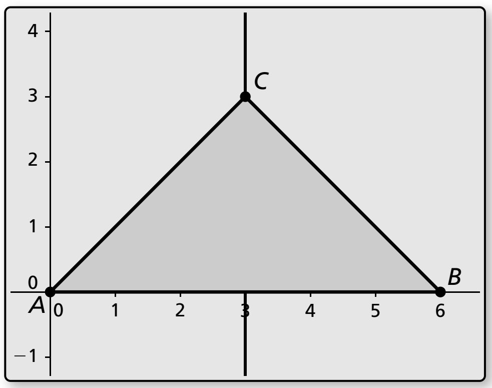
Work with a partner.

a. Use dynamic geometry software to draw $\overline{AB}$ with endpoints $A(0, 0)$ and $B(6, 0)$.

b. Draw the vertical line $x = 3$.

c. Plot the point $C(3, 3)$ and draw $\triangle ABC$. Then use your drawing to prove that $\triangle ABC$ is an isosceles right triangle.

2 EXPLORATION: Writing a Coordinate Proof (continued)



Sample
Points
$A(0, 0)$
$B(6, 0)$
$C(3, 3)$
Segments
$AB = 6$
$BC = 4.24$
$AC = 4.24$
Line
$x = 3$

d. Change the coordinates of C so that C lies below the x-axis and $\triangle ABC$ is an isosceles right triangle.

e. Write a coordinate proof to show that if C lies on the line $x = 3$ and $\triangle ABC$ is an isosceles right triangle, then C must be the point (3, 3) or the point found in part (d).

Communicate Your Answer

3. How can you use a coordinate plane to write a proof?

4. Write a coordinate proof to prove that $\triangle ABC$ with vertices $A(0, 0)$, $B(6, 0)$, and $C\left(3, 3\sqrt{3}\right)$ is an equilateral triangle.

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11.3 Practice
For use after Lesson 11.3

Notes:

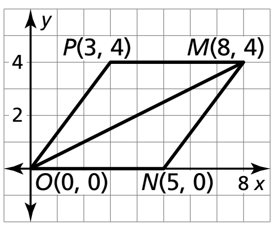
Worked-Out Examples

Example #1

Write a plan for the proof.

Given Coordinates of vertices of $\triangle OPM$ and $\triangle ONM$

Prove $\triangle OPM$ and $\triangle ONM$ are isosceles triangles.

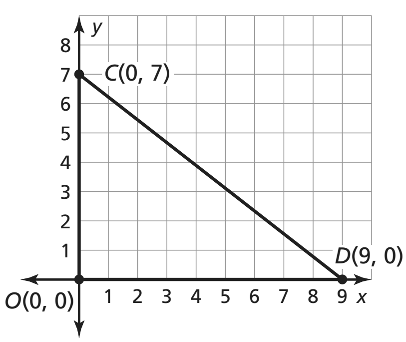


Find the lengths of $\overline{OP}$, $\overline{PM}$, $\overline{MN}$, and $\overline{NO}$ to show that $\overline{OP} \cong \overline{PM}$ and $\overline{MN} \cong \overline{NO}$.

Example #2

Place the figure in a coordinate plane and find the indicated length.

A right triangle with the leg lengths of 7 and 9 units; Find the length of the hypotenuse



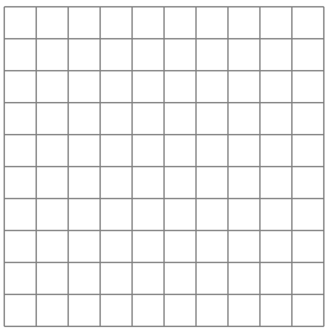
$$CD = \sqrt{(0-9)^2 + (7-0)^2} = \sqrt{(-9)^2 + 7^2}$$
$$= \sqrt{81 + 49} = \sqrt{130} \approx 11.4$$

The length of the hypotenuse is about 11.4 units.

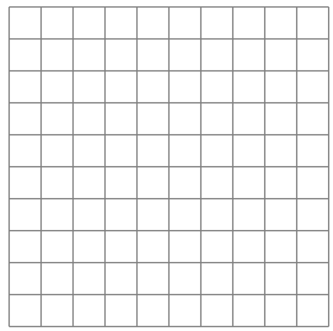
Practice A

In Exercises 1 and 2, place the figure in a coordinate plane in a convenient way. Assign coordinates to each vertex. Explain the advantages of your placement.

1. an obtuse triangle with height of 3 units and base of 2 units



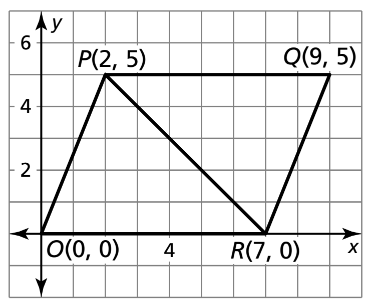
2. a rectangle with length of $2w$



In Exercises 3 and 4, write a plan for the proof.

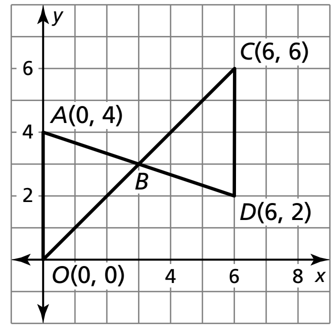
3. Given Coordinates of vertices of $\triangle OPR$ and $\triangle QRP$

Proof $\triangle OPR \cong \triangle QRP$

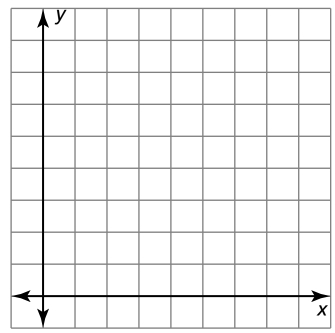


4. **Given** Coordinates of vertices of $\triangle OAB$ and $\triangle CDB$

Prove B is the midpoint of $\overline{AD}$ and $\overline{OC}$.



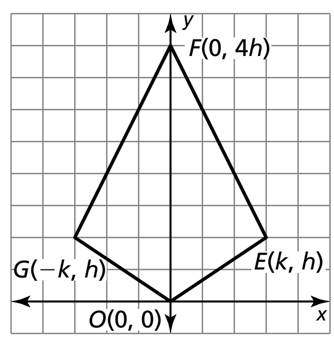
5. Graph the triangle with vertices $A(0, 0)$, $B(3m, m)$, and $C(0, 3m)$. Find the length and the slope of each side of the triangle. Then find the coordinates of the midpoint of each side. Is the triangle a right triangle? isosceles? Explain. (Assume all variables are positive.)



6. Write a coordinate proof.

Given Coordinates of vertices of $\triangle OEF$ and $\triangle OGF$

Prove $\triangle OEF \cong \triangle OGF$



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Practice B

In Exercises 1–3, place the figure in a coordinate plane in a convenient way. Assign coordinates to each vertex. Explain the advantages of your placement.

1. a rectangle twice as long as it is wide

2. a right triangle with a leg length of 3 units and a hypotenuse with a positive slope

3. an obtuse scalene triangle

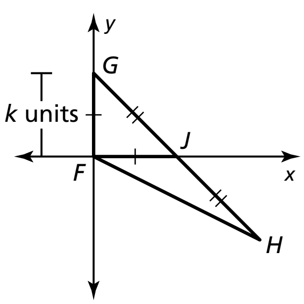
In Exercises 4 and 5, graph the triangle with the given vertices. Find the length and the slope of each side of the triangle. Then find the coordinates of the midpoint of each side. Is the triangle a right triangle? isosceles? Explain.

4. $J(0, 0), K(a, b), L(2a, 0)$

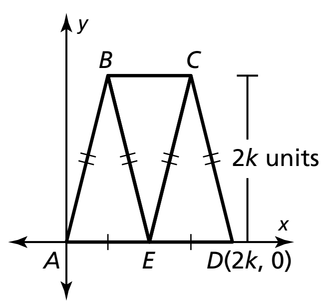
5. $P(0, 0), Q(5a, 0), R(8a, 4a)$

In Exercises 6 and 7, find the coordinates of any unlabeled vertices. Then find the indicated lengths.

6. Find *GH* and *FH*.



7. Find *BC* and *CD*.



8. The vertices of a quadrilateral are given by the coordinates $W(3, 5), X(5, 0), Y(-3, -4)$, and $Z(-5, 1)$. Is the quadrilateral a parallelogram? a trapezoid? Explain your reasoning.

9. Write a coordinate proof for the following statement.

   Any $\triangle ABC$ formed so that vertex *C* is on the perpendicular bisector of $\overline{AB}$ is an isosceles triangle.