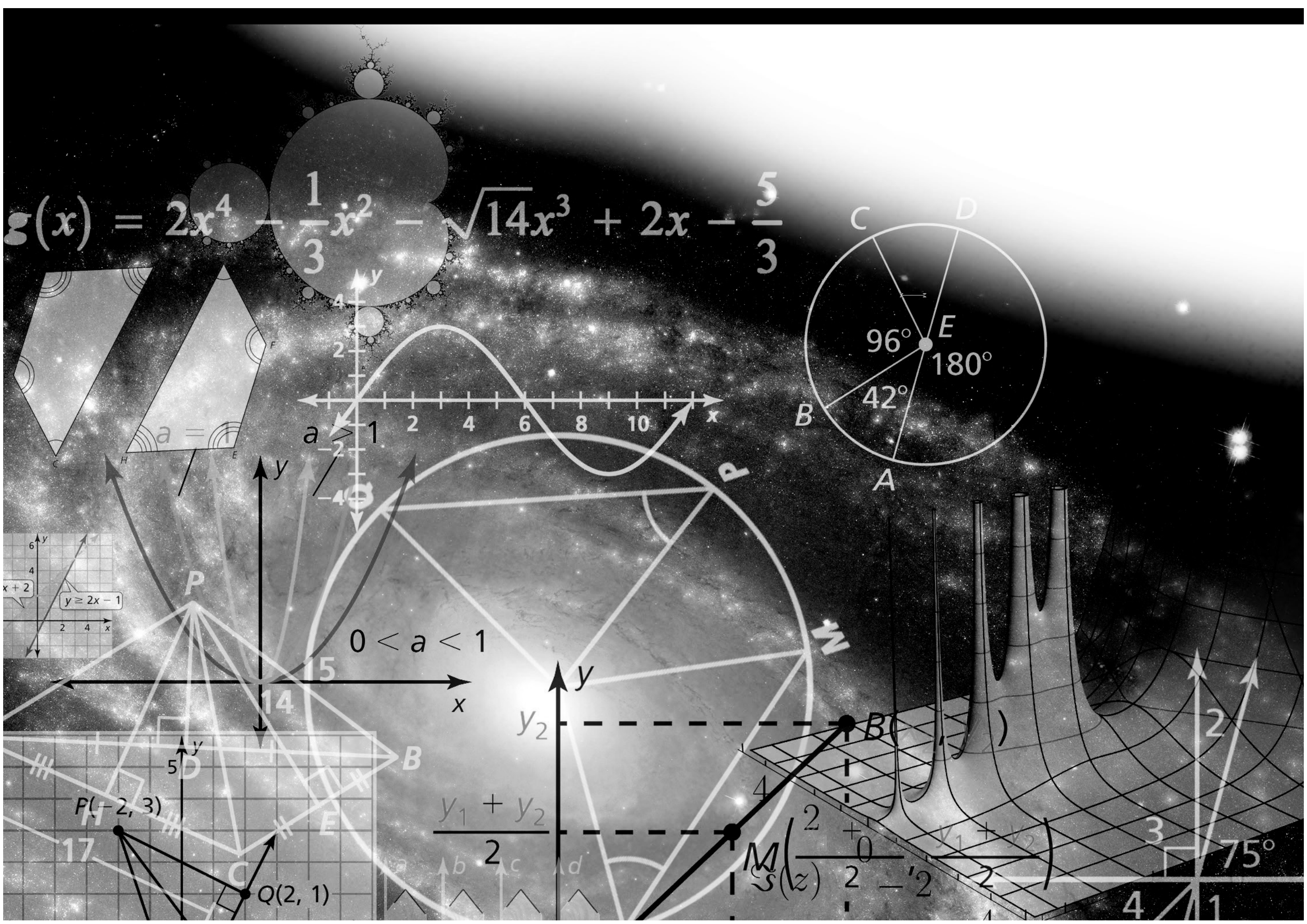
CHAPTER 11

Geometry



Name____ Date __________

Chapter 11 Maintaining Mathematical Proficiency

Simplify the expression.

1. $|-3 + (-1)| =$

2. $|10 - 11| =$

3. $|-6 + 8| =$

4. $|9 - (-1)| =$

5. $|-12 - (-8)| =$

6. $|-15 - 7| =$

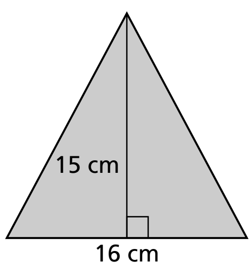
7. $|-12 + 3| =$

8. $|5 + (-15)| =$

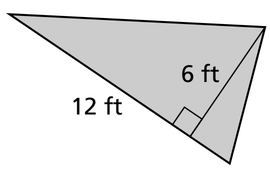
9. $|1 - 12| =$

Find the area of the triangle.

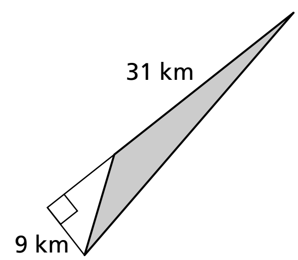
10.



11.



12.



Name ______________________________ Date __________

11.1 Using Midpoint and Distance Formulas

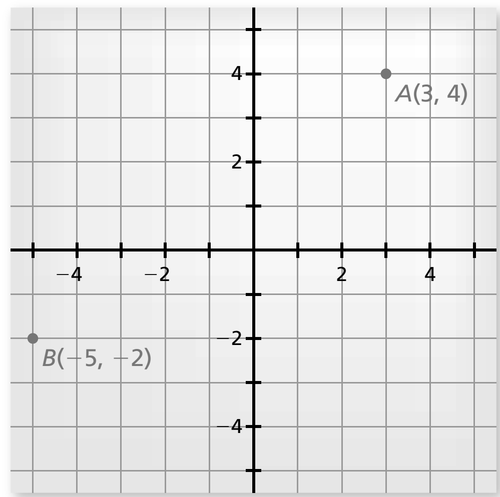
For use with Exploration 11.1

Essential Question How can you find the midpoint and length of a line segment in a coordinate plane?

1 EXPLORATION: Finding the Midpoint of a Line Segment

Work with a partner. Use centimeter graph paper.

a. Graph $\overline{AB}$, where the points A and B are as shown.



b. Explain how to *bisect* $\overline{AB}$, that is, to divide $\overline{AB}$ into two congruent line segments. Then bisect $\overline{AB}$ and use the result to find the *midpoint* M of $\overline{AB}$.

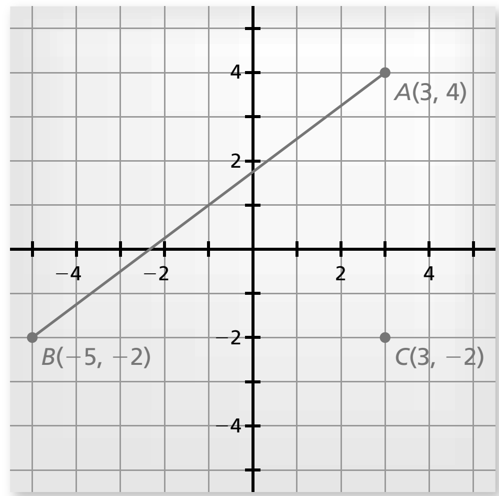
c. What are the coordinates of the midpoint M?

d. Compare the x-coordinates of A, B, and M. Compare the y-coordinates of A, B, and M. How are the coordinates of the midpoint M related to the coordinates of A and B?

Name______________________________ Date__________

2 EXPLORATION: Finding the Length of a Line Segment

Work with a partner. Use centimeter graph paper.



a. Add point C to your graph as shown.

b. Use the Pythagorean Theorem to find the length of $\overline{AB}$.

c. Use a centimeter ruler to verify the length you found in part (b).

d. Use the Pythagorean Theorem and point M from Exploration 1 to find the lengths of $\overline{AM}$ and $\overline{MB}$. What can you conclude?

Communicate Your Answer

3. How can you find the midpoint and length of a line segment in a coordinate plane?

4. Find the coordinates of the midpoint M and the length of the line segment whose endpoints are given.

a. $D(-10, -4), E(14, 6)$ **b.** $F(-4, 8), G(9, 0)$

Name ______________________________ Date ________

11.1 Practice
For use after Lesson 11.1

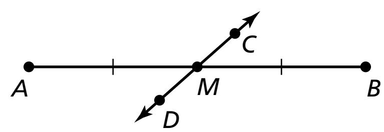
Core Concepts

Midpoints and Segment Bisectors

The **midpoint** of a segment is the point that divides the segment into two congruent segments.

M is the midpoint of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.



$\overleftrightarrow{CD}$ is a segment bisector of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

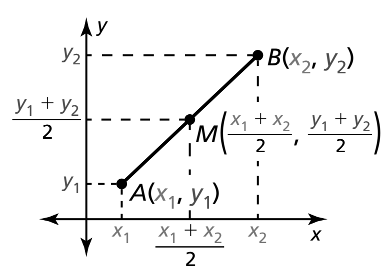
Notes:

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the x-coordinates and of the y-coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of $\overline{AB}$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



Notes:

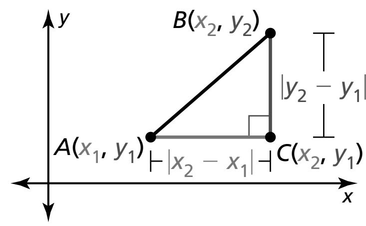
Name______________________________ Date__________

11.1 Practice (continued)

The Distance Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

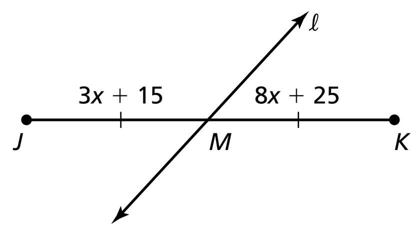


Notes:

Worked-Out Examples

Example #1

Identify the segment bisector of $\overline{JK}$. Then find JM.



Line ℓ bisects $\overline{JK}$ at point M.

$$\begin{aligned} JM &= MK \\ 3x + 15 &= 8x + 25 \\ 15 &= 5x + 25 \\ -10 &= 5x \\ -2 &= x \end{aligned} \qquad \begin{aligned} JM &= 3(-2) + 15 \\ &= -6 + 15 \\ &= 9 \end{aligned}$$

Example #2

The midpoint M and one endpoint of $\overline{GH}$ are given. Find the coordinates of the other endpoint.

$M(4, 3)$, $G(5, -6)$, $H(x, y)$

$$(4, 3) = \left(\frac{5 + x}{2}, \frac{-6 + y}{2}\right)$$

$$\begin{aligned} 4 &= \frac{5 + x}{2} \\ 8 &= 5 + x \\ 3 &= x \end{aligned} \qquad \begin{aligned} 3 &= \frac{-6 + y}{2} \\ 6 &= -6 + y \\ 12 &= y \end{aligned}$$

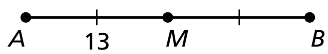
$H(3, 12)$

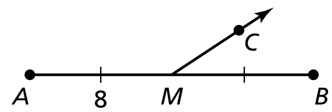
Name ______________________________ Date __________

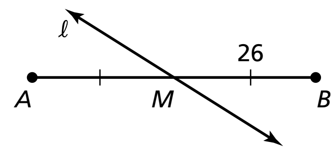
11.1 **Practice** (continued)

Practice A

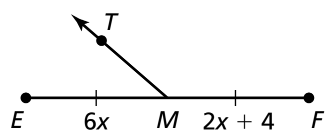
In Exercises 1–3, identify the segment bisector of $\overline{AB}$. Then find AB.

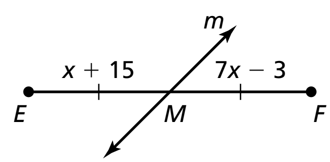
1.


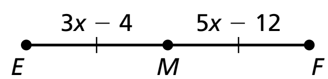
2.


3.


In Exercises 4-6, identify the segment bisector of $\overline{EF}$. Then find EF.

4.


5.


6.


In Exercises 7–9, the endpoints of $\overline{PQ}$ are given. Find the coordinates of the midpoint M.

7. $P(-4, 3)$ and $Q(0, 5)$

8. $P(-2, 7)$ and $Q(10, -3)$

9. $P(3, -15)$ and $Q(9, -3)$

In Exercises 10–12, the midpoint M and one endpoint of $\overline{JK}$ are given. Find the coordinates of the other endpoint.

10. $J(7, 2)$ and $M(1, -2)$

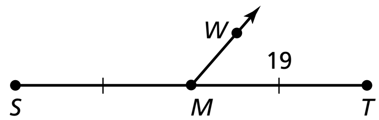
11. $J(5, -2)$ and $M(0, -1)$

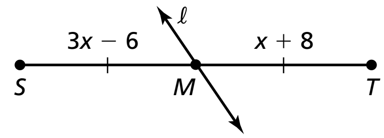
12. $J(2, 16)$ and $M\left(-\frac{9}{2}, 7\right)$

Name ____ Date _________

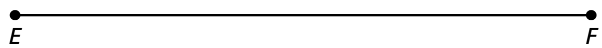
Practice B

In Exercises 1 and 2, identify the bisector of $\overline{ST}$. Then find ST.

1. 

2. 

Copy the segment and construct a segment bisector by paper folding. Then label the midpoint *M*.

3. 

In Exercises 4 and 5, the endpoints of $\overline{LN}$ are given. Find the coordinates of the midpoint *M*.

4. $L(2, 1)$ and $N(2, 13)$

5. $L(-6, 0)$ and $N(6, 6)$

In Exercises 6 and 7, the midpoint *M* and one endpoint of $\overline{CD}$ are given. Find the coordinates of the other endpoint.

6. $M(1, 2)$ and $C(-1, 4)$

7. $M(3, 7)$ and $D(1, 1)$

In Exercises 8 and 9, find the distance between the two points.

8. $A(1, 7)$ and $B(4, 6)$

9. $G(-1, -5)$ and $H(3, -8)$

10. Your friend draws a square and one diagonal connecting its opposite vertices. Your friend believes that the diagonal is the same length as one side of the square. Do you agree? Explain your reasoning.

11. Is it possible for a segment to have more than one bisector? Explain your reasoning.

12. You walk 2 miles from your house to the park and 4.5 miles from the park to the lake. Then you return home along a straight path from the lake. How many miles do you walk from the lake back to your house? What is the total distance you walk?