Name_____ Date _____

8.7 Characteristics of Quadratic Functions

For use with Exploration 8.7

Essential Question What type of symmetry does the graph of $f(x) = a(x - h)^2 + k$ have and how can you describe this symmetry?

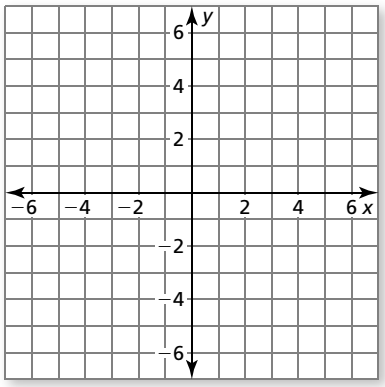
1 EXPLORATION: Parabolas and Symmetry

Work with a partner.

a. Complete the table. Then use the values in the table to sketch the graph of the function $f(x) = \frac{1}{2}x^2 - 2x - 2$ on graph paper.

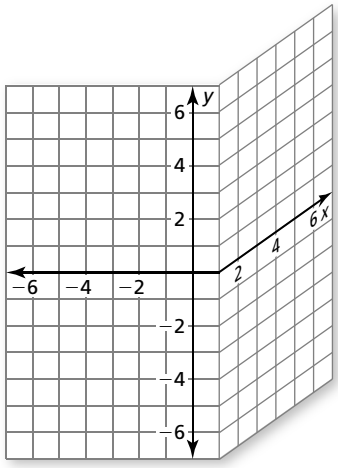
x	−2	−1	0	1	2
$f(x)$					

x	3	4	5	6
$f(x)$				



b. Use the results in part (a) to identify the vertex of the parabola.

c. Find a vertical line on your graph paper so that when you fold the paper, the left portion of the graph coincides with the right portion of the graph. What is the equation of this line? How does it relate to the vertex?



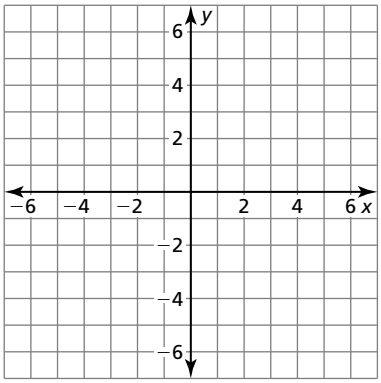
d. Show that the vertex form $f(x) = \frac{1}{2}(x - 2)^2 - 4$ is equivalent to the function given in part (a).

2 EXPLORATION: Parabolas and Symmetry

Work with a partner. Repeat Exploration 1 for the function given by $f(x) = -\frac{1}{3}x^2 + 2x + 3 = -\frac{1}{3}(x - 3)^2 + 6$.

x	−2	−1	0	1	2
$f(x)$					

x	3	4	5	6
$f(x)$				



Communicate Your Answer

3. What type of symmetry does the graph of $f(x) = a(x - h)^2 + k$ have and how can you describe this symmetry?

4. Describe the symmetry of each graph. Then use a graphing calculator to verify your answer.

a. $f(x) = -(x - 1)^2 + 4$ **b.** $f(x) = (x + 1)^2 - 2$ **c.** $f(x) = 2(x - 3)^2 + 1$

d. $f(x) = \frac{1}{2}(x + 2)^2$ **e.** $f(x) = -2x^2 + 3$ **f.** $f(x) = 3(x - 5)^2 + 2$

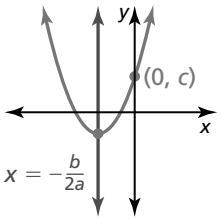
Name_____ Date _____

8.7 Practice
For use after Lesson 8.7

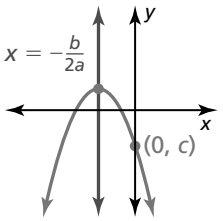
Core Concepts

Properties of the Graph of $f(x) = ax^2 + bx + c$

$y = ax^2 + bx + c, a > 0$



$y = ax^2 + bx + c, a < 0$



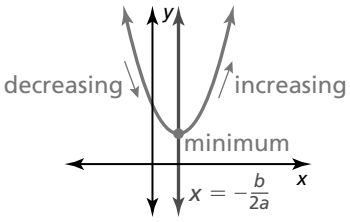
- The parabola opens up when $a > 0$ and open down when $a < 0$.
- The graph is narrower than the graph of $f(x) = x^2$ when $|a| > 1$ and wider when $|a| < 1$.
- The axis of symmetry is $x = -\frac{b}{2a}$ and the vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
- The y-intercept is c. So, the point $(0, c)$ is on the parabola.

Notes:

Minimum and Maximum Values

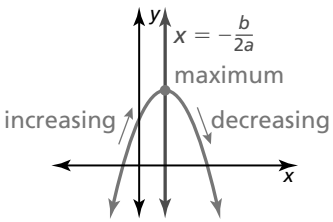
For the quadratic function $f(x) = ax^2 + bx + c$, the y-coordinate of the vertex is the **minimum value** of the function when $a > 0$ and the **maximum value** when $a < 0$.

$a > 0$



- Minimum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \geq f\left(-\frac{b}{2a}\right)$
- Decreasing to the left of $x = -\frac{b}{2a}$
- Increasing to the right of $x = -\frac{b}{2a}$

$a < 0$



- Maximum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \leq f\left(-\frac{b}{2a}\right)$
- Increasing to the left of $x = -\frac{b}{2a}$
- Decreasing to the right of $x = -\frac{b}{2a}$

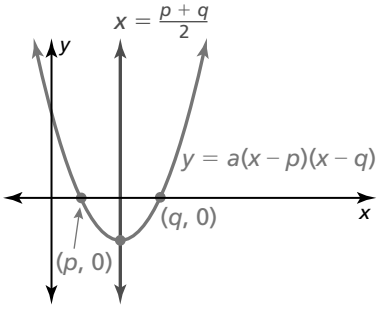
Name _____ Date _____

8.7 Practice (continued)

Notes:

Properties of the Graph of $f(x) = a(x - p)(x - q)$

- Because $f(p) = 0$ and $f(q) = 0$, p and q are the x-intercepts of the graph of the function.
- The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$. So, the axis of symmetry is $x = \frac{p + q}{2}$.
- The parabola opens up when $a > 0$ and opens down when $a < 0$.



Notes:

Worked-Out Examples

Example #1

Graph the function. Label the vertex and axis of symmetry.

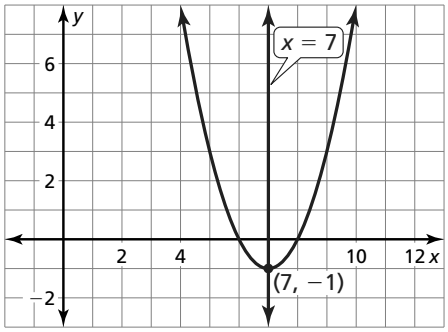
$y = (x - 7)^2 - 1$

Identify the constants $a = 1$, $h = 7$, and $k = -1$. Plot the vertex $(h, k) = (7, -1)$ and draw the axis of symmetry $x = 7$. Evaluate the function for two values of x.

$x = 5$: $y = (5 - 7)^2 - 1 = 3$

$x = 6$: $y = (6 - 7)^2 - 1 = 0$

Plot the points (5, 3), (6, 0), and their reflections in the axis of symmetry. Draw a parabola through the plotted points.



Example #2

Graph the function. Label the vertex and axis of symmetry

$y = 3x^2 - 6x + 4$

Identify the coefficients $a = 3$, $b = 6$, and $c = 4$. Because $a > 0$, the parabola opens up. Find the vertex. First calculate the x-coordinate.

$$x = -\frac{b}{2a} = \frac{-6}{2(3)} = 1$$

Then find the y-coordinate of the vertex.

$y = 3(1)^2 - 6(1) + 4 = 1$

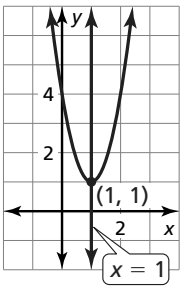
So, the vertex is (1, 1). Plot this point. Draw the axis of symmetry. Identify the y-intercept, c, which is 4. Plot the point (0, 4) and its reflection in the axis of symmetry, (2, 4). Evaluate the function for another value of x, such as $x = 3$.

Name_____ Date _____

8.7 Practice (continued)

$y = 3(3)^2 - 6(3) + 4 = 13$

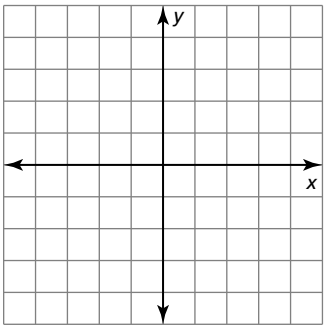
Plot the point (3, 13) and its reflection in the axis of symmetry, $(-1, 13)$. Draw a parabola through the plotted points.

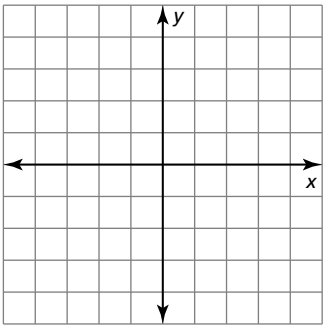


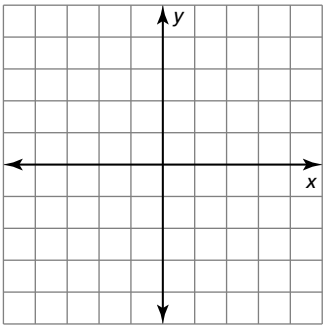
Practice A

In Exercises 1–3, graph the function. Label the vertex and axis of symmetry. Find the minimum or maximum value of the function. Describe the domain and range of the function, and where the function is increasing and decreasing.

1. $f(x) = (x + 1)^2$ **2.** $y = -2(x - 4)^2 - 5$ **3.** $t(x) = \frac{3}{2}x^2 - 3x - 1$

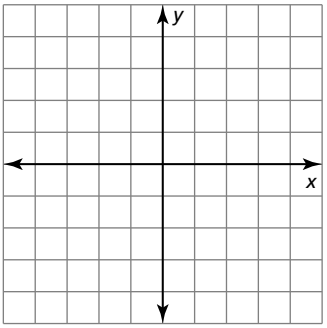


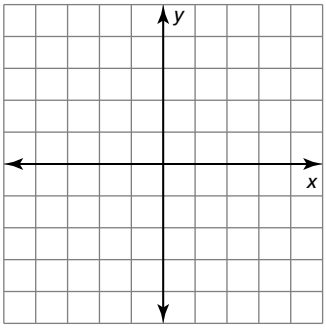




In Exercises 4 and 5, graph the function. Label the *x*-intercept(s), vertex, and axis of symmetry.

4. $f(x) = 4(x + 4)(x - 3)$ **5.** $f(x) = -7x(x - 6)$





6. A softball player hits a ball whose path is modeled by $f(x) = -0.0005x^2 + 0.2127x + 3$, where x is the distance from home plate (in feet) and y is the height of the ball above the ground (in feet). What is the highest point this ball will reach? If the ball was hit to center field which has an 8 foot fence located 410 feet from home plate, was this hit a home run? Explain.

Name _____ Date _____

Practice B

In Exercises 1–12, graph the function. Label the vertex and axis of symmetry.

1. $f(x) = -3(x - 2)^2 - 4$

2. $f(x) = 3(x + 1)^2 + 5$

3. $g(x) = -\frac{1}{2}(x + 3)^2 + 2$

4. $h(x) = \frac{1}{2}(x - 2)^2 - 1$

5. $y = 0.6(x - 2)^2$

6. $f(x) = 0.25x^2 - 1$

7. $y = -x^2 + 8$

8. $y = 7x^2 + 2$

9. $y = 1.5x^2 - 6x + 3$

10. $f(x) = 0.5x^2 + 3x - 1$

11. $y = \frac{5}{2}x^2 - 5x + 1$

12. $f(x) = -\frac{3}{2}x^2 - 6x - 4$

13. A quadratic function is decreasing to the left of $x = 3$ and increasing to the right of $x = 3$. Will the vertex be the highest or lowest point on the graph of the parabola? Explain.

14. The graph of which function has the same axis of symmetry as the graph of $y = 2x^2 - 8x + 3$? Explain your reasoning.

A. $y = -4x^2 + 16x - 5$

B. $y = 2x^2 + 8x + 7$

C. $y = 3x^2 - 6x + 7$

D. $y = -6x^2 + 10x - 1$

In Exercises 15–18, find the minimum or maximum value of the function. Describe the domain and range of the function, and where the function is increasing and decreasing.

15. $y = 3x^2 + 12$

16. $y = -x^2 - 6x$

17. $y = -\frac{1}{3}x^2 - 2x + 3$

18. $f(x) = \frac{1}{2}x^2 + 3x + 7$

19. The height of a bridge is given by $y = -3x^2 + x$, where y is the height of the bridge (in miles) and x is the number of miles from the base of the bridge.

a. How far from the base of the bridge does the maximum height occur?

b. What is the maximum height of the bridge?