

8.4

Graphing $f(x) = a(x - h)^2 + k$

For use with Exploration 8.4

Essential Question How can you describe the graph of $f(x) = a(x - h)^2$?

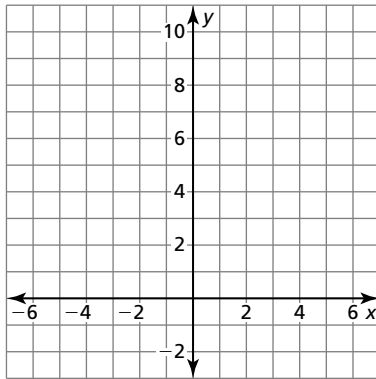
1 EXPLORATION: Graphing $y = a(x - h)^2$ When $h > 0$

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

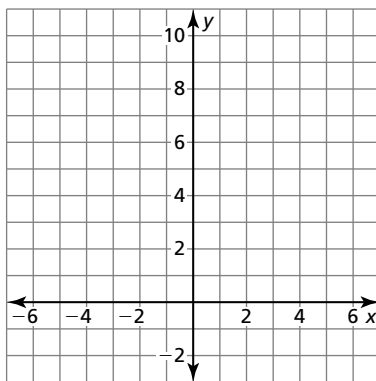
Work with a partner. Sketch the graphs of the functions in the same coordinate plane.

How does the value of h affect the graph of $y = a(x - h)^2$?

a. $f(x) = x^2$ and $g(x) = (x - 2)^2$



b. $f(x) = 2x^2$ and $g(x) = 2(x - 2)^2$

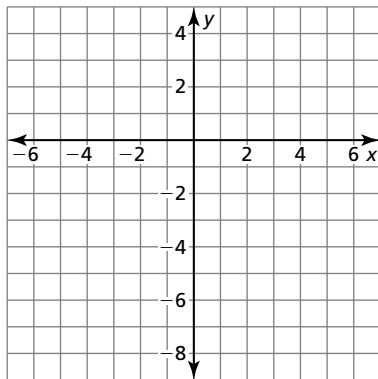


8.4 Graphing $f(x) = a(x - h)^2 + k$ (continued)**2** **EXPLORATION:** Graphing $y = a(x - h)^2$ When $h < 0$

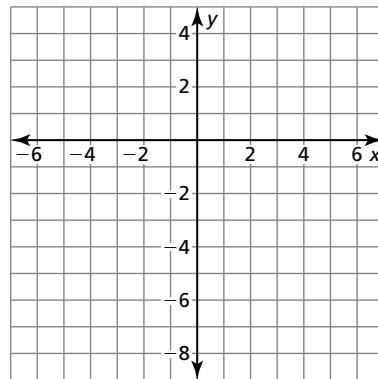
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Sketch the graphs of the functions in the same coordinate plane. How does the value of h affect the graph of $y = a(x - h)^2$?

a. $f(x) = -x^2$ and $g(x) = -(x + 2)^2$



b. $f(x) = -2x^2$ and $g(x) = -2(x + 2)^2$

**Communicate Your Answer**

- How can you describe the graph of $f(x) = a(x - h)^2$?
- Without graphing, describe the graph of each function. Use a graphing calculator to check your answer.
 - $y = (x - 3)^2$
 - $y = (x + 3)^2$
 - $y = -(x - 3)^2$

8.4

Practice
For use after Lesson 8.4

Core Concepts

Even and Odd Functions

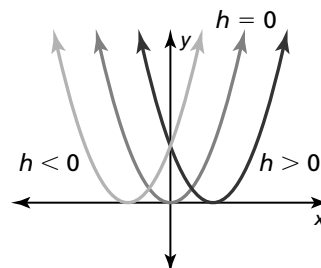
A function $y = f(x)$ is **even** when $f(-x) = f(x)$ for each x in the domain of f . The graph of an even function is symmetric about the y -axis.

A function $y = f(x)$ is **odd** when $f(-x) = -f(x)$ for each x in the domain of f . The graph of an odd function is symmetric about the origin. A graph is *symmetric about the origin* when it looks the same after reflections in the x -axis and then in the y -axis.

Notes:

Graphing $f(x) = a(x - h)^2$

- When $h > 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation h units right of the graph $f(x) = ax^2$.
- When $h < 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation $|h|$ units left of the graph of $f(x) = ax^2$.

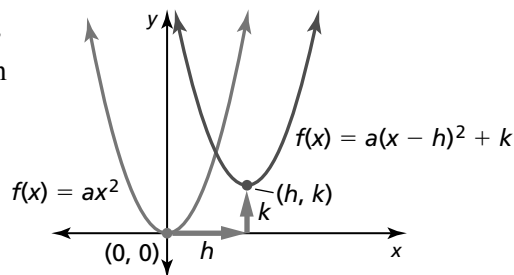


The vertex of the graph of $f(x) = a(x - h)^2$ is $(h, 0)$, and the axis of symmetry is $x = h$.

Notes:

Graphing $f(x) = a(x - h)^2 + k$

The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$. The graph of $f(x) = a(x - h)^2 + k$ is a translation h units horizontally and k units vertically of the graph of $f(x) = ax^2$.



The vertex of the graph of $f(x) = a(x - h)^2 + k$ is (h, k) , and the axis of symmetry is $x = h$.

Notes:

8.4 Practice (continued)**Worked-Out Examples****Example #1**

Determine whether the function is even, odd, or neither.

$$f(x) = 4x + 3$$

$$\begin{aligned} f(-x) &= 4(-x) + 3 \\ &= -4x + 3 \end{aligned}$$

Because $f(x) = 4x + 3$ and $-f(x) = -4x - 3$, you can conclude that $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$. So, the function is neither odd nor even.

Example #2

Find the vertex and the axis of symmetry of the graph of the function.

$$f(x) = 3(x + 1)^2$$

For $f(x) = 3(x + 1)^2$, because $h = -1$, the axis of symmetry is $x = -1$, and the vertex is $(-1, 0)$.

Practice A

In Exercises 1–4, determine whether the function is *even*, *odd*, or *neither*.

1. $f(x) = 5x$

2. $f(x) = -4x^2$

3. $h(x) = \frac{1}{2}x^2$

4. $f(x) = -3x^2 + 2x + 1$

In Exercises 5–8, find the vertex and the axis of symmetry of the graph of the function.

5. $f(x) = 5(x - 2)^2$

6. $f(x) = -4(x + 8)^2$

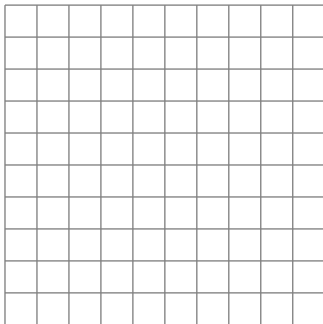
8.4 Practice (continued)

7. $p(x) = -\frac{1}{2}(x - 1)^2 + 4$

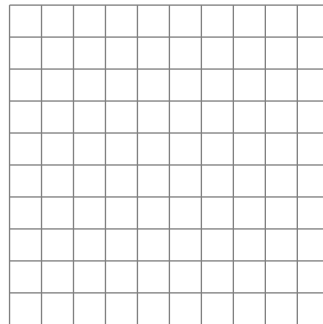
8. $g(x) = -(x + 1)^2 - 5$

In Exercises 9 and 10, graph the function. Compare the graph to the graph of $f(x) = x^2$.

9. $m(x) = 3(x + 2)^2$

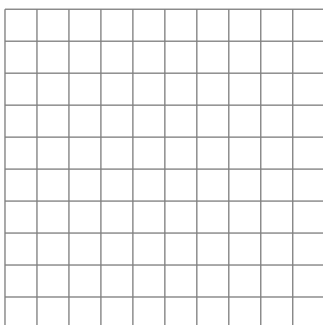


10. $g(x) = -\frac{1}{4}(x - 6)^2 + 4$

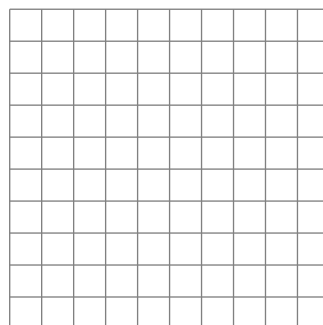


In Exercises 11 and 12, graph g .

11. $f(x) = 3(x + 1)^2 - 1$; $g(x) = f(x + 2)$



12. $f(x) = \frac{1}{2}(x - 3)^2 - 5$; $g(x) = -f(x)$



Practice B

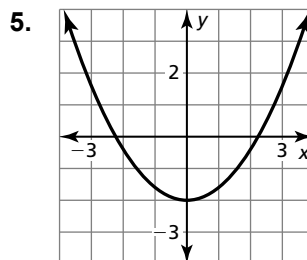
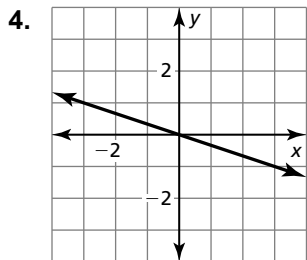
In Exercises 1–3, determine whether the function is *even*, *odd*, or *neither*.

1. $f(x) = 3x^2 + 2x$

2. $g(x) = \frac{2}{3}x$

3. $h(x) = \frac{1}{3}x^2 - 2$

In Exercises 4 and 5, determine whether the function represented by the graph is *even*, *odd*, or *neither*.



In Exercises 6–8, find the vertex and the axis of symmetry of the graph of the function.

6. $f(x) = -\frac{1}{3}(x + 6)^2$

7. $f(x) = 9(x - 4)^2$

8. $y = -10(x + 9)^2$

In Exercises 9–11, graph the function. Compare the graph to the graph of $f(x) = x^2$.

9. $g(x) = 4(x + 2)^2$

10. $g(x) = \frac{1}{3}(x - 5)^2$

11. $g(x) = \frac{1}{6}(x - 1)^2$

In Exercises 12–14, find the vertex and the axis of symmetry of the graph of the function.

12. $y = 6(x - 4)^2 - 3$

13. $f(x) = -4(x + 1)^2 + 5$

14. $y = -(x + 3)^2 - 2$

In Exercises 15 and 16, graph the function. Compare the graph to the graph of $f(x) = x^2$.

15. $g(x) = 3(x + 2)^2 - 1$

16. $g(x) = -\frac{1}{2}(x - 1)^2 + 3$

In Exercises 17 and 18, rewrite the quadratic function in vertex form.

17. $y = 5x^2 - 10x + 2$

18. $f(x) = -2x^2 + 8x + 5$

19. The graph of $y = x^2$ is reflected in the x -axis and translated 3 units right and 2 units up. Write an equation for the function in vertex form and in standard form. Describe advantages of writing the function in each form.