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8.3 Graphing $f(x) = ax^2 + bx + c$

For use with Exploration 8.3

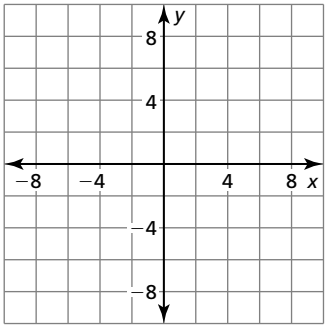
Essential Question How can you find the vertex of the graph of $f(x) = ax^2 + bx + c$?

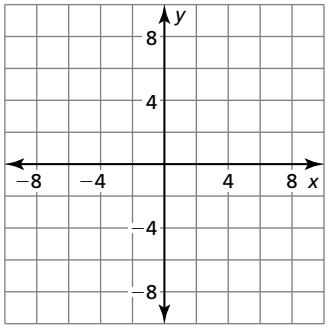
1 EXPLORATION: Comparing x-Intercepts with the Vertex

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Sketch the graphs of $y = 2x^2 - 8x$ and $y = 2x^2 - 8x + 6$.





b. What do you notice about the x-coordinate of the vertex of each graph?

c. Use the graph of $y = 2x^2 - 8x$ to find its x-intercepts. Verify your answer by solving $0 = 2x^2 - 8x$.

d. Compare the value of the x-coordinate of the vertex with the values of the x-intercepts.

8.3 Graphing $f(x) = ax^2 + bx + c$ (continued)

2 EXPLORATION: Finding x-Intercepts

Work with a partner.

a. Solve $0 = ax^2 + bx$ for x by factoring.

b. What are the x-intercepts of the graph of $y = ax^2 + bx$?

c. Complete the table to verify your answer.

x	$y = ax^2 + bx$
0	
$-\frac{b}{a}$	

3 EXPLORATION: Deductive Reasoning

Work with a partner. Complete the following logical argument.

The x-intercepts of the graph of $y = ax^2 + bx$ are 0 and $-\frac{b}{a}$.

The vertex of the graph of $y = ax^2 + bx$ occurs when $x =$ _____.

The vertices of the graphs of $y = ax^2 + bx$ and $y = ax^2 + bx + c$ have the same x-coordinate.

The vertex of the graph of $y = ax^2 + bx + c$ occurs when $x =$ _____.

Communicate Your Answer

4. How can you find the vertex of the graph of $f(x) = ax^2 + bx + c$?

5. Without graphing, find the vertex of the graph of $f(x) = x^2 - 4x + 3$. Check your result by graphing.

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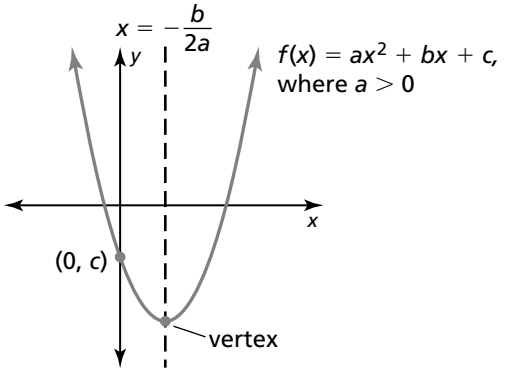
8.3 Practice
For use after Lesson 8.3

Core Concepts

Graphing $f(x) = ax^2 + bx + c$

- The graph opens up when $a > 0$, and the graph opens down when $a < 0$.
- The y-intercept is c.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.
- The axis of symmetry is $x = -\frac{b}{2a}$.

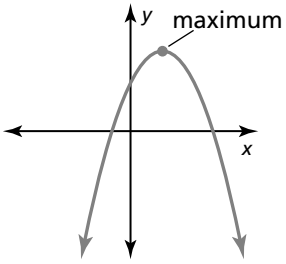


Notes:

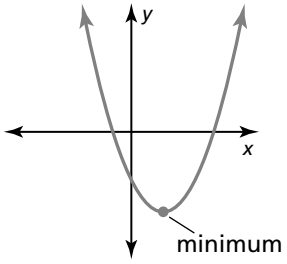
Maximum and Minimum Values

The y-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$ is the **maximum value** of the function when $a < 0$ or the **minimum value** of the function when $a > 0$.

$f(x) = ax^2 + bx + c, a < 0$



$f(x) = ax^2 + bx + c, a > 0$



Notes:

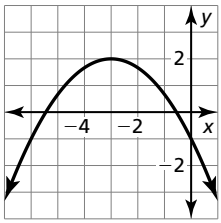
Name_____ Date _____

8.3 Practice (continued)

Worked-Out Examples

Example #1

Find the vertex, the axis of symmetry, and the y-intercept of the graph.



The vertex is $(-3, 2)$. The axis of symmetry is $x = -3$. The y-intercept of the graph is -1.

Example #2

Find (a) the axis of symmetry and (b) the vertex of the graph of the function.

$y = -9x^2 - 18x - 1$

a. $x = -\frac{b}{2a} = -\frac{(-18)}{2(-9)} = \frac{18}{-18} = -1$

The axis of symmetry is $x = -1$.

b. $y = -9x^2 - 18x - 1$

$y = -9(-1)^2 - 18(-1) - 1$

$= -9(1) + 18 - 1$

$= -9 + 18 - 1$

$= 9 - 1$

$= 8$

The vertex is $(-1, 8)$.

Practice A

In Exercises 1–4, find (a) the axis of symmetry and (b) the vertex of the graph of the function.

1. $f(x) = x^2 - 10x + 2$

2. $y = -4x^2 + 16x$

3. $y = -2x^2 - 8x + 5$

4. $f(x) = -3x^2 + 6x + 1$

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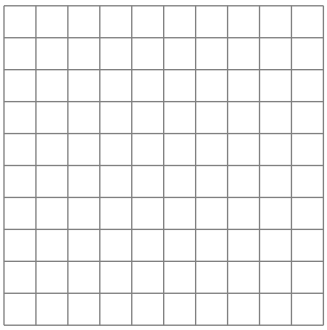
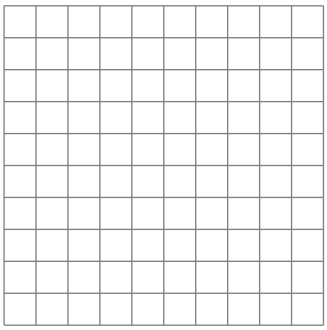
8.3 Practice (continued)

In Exercises 5–7, graph the function. Describe the domain and range.

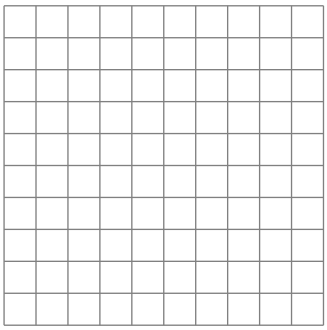
5. $f(x) = 3x^2 + 6x + 2$



6. $y = 2x^2 - 8x - 1$



7. $y = -\frac{1}{5}x^2 - x + 5$



In Exercises 8–13, tell whether the function has a minimum value or a maximum value. Then find the value.

8. $y = -\frac{1}{2}x^2 - 5x + 2$

9. $y = 8x^2 + 16x - 2$

10. $y = -x^2 - 4x - 7$

11. $y = -7x^2 + 7x + 5$

12. $y = 9x^2 + 6x + 4$

13. $y = -\frac{1}{4}x^2 + x - 6$

14. The function $h = -16t^2 + 250t$ represents the height h (in feet) of a rocket t seconds after it is launched. The rocket explodes at its highest point.

a. When does the rocket explode?

b. At what height does the rocket explode?

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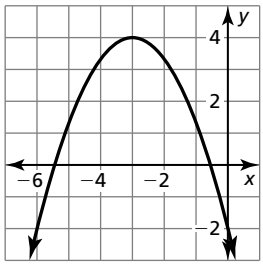
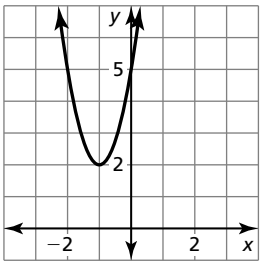
Practice B

In Exercises 1 and 2, find the vertex, the axis of symmetry, and the *y*-intercept of the graph.

1.



2.



In Exercises 3–6, find (a) the axis of symmetry and (b) the vertex of the graph of the function.

3. $f(x) = 4x^2 + 12x$

4. $y = -5x^2 - 20x + 4$

5. $y = -8x^2 + 24x + 13$

6. $f(x) = \frac{2}{3}x^2 - 6x + 15$

In Exercises 7–10, graph the function. Describe the domain and range.

7. $f(x) = 4x^2 + 8x + 11$

8. $y = -6x^2 - 12x - 7$

9. $y = \frac{1}{2}x^2 - 8x + 3$

10. $f(x) = -\frac{2}{3}x^2 + 4x + 2$

11. Describe and correct the error in finding the vertex of the graph of $y = x^2 + 6x + 2$.

$$\times \quad x = -\frac{b}{2a} = -\frac{6}{2(1)} = -3$$

So, the vertex is $(-3, 2)$.

In Exercises 12 and 13, tell whether the function has a minimum value or a maximum value. Then find the value.

12. $f(x) = -6x^2 + 24x - 5$

13. $y = \frac{1}{3}x^2 + 8x - 1$

In Exercises 14 and 15, use the *minimum* or *maximum* feature of a graphing calculator to approximate the vertex of the graph of the function.

14. $y = -2.1x^2 + \pi x + 3$

15. $y = 1.25x^2 - 2^{3/4}x + 3$