CHAPTER 8 Graphing Quadratic Functions

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Graph the linear equation.

1.
$$y = 4x - 5$$



3. $y = \frac{1}{2}x + 3$





4. y = -x + 2

2. y = -2x + 3

Evaluate the expression when x = -4.

5. $2x^2 + 8$ **6.** $-x^2 + 3x - 4$ **7.** $-3x^2 - 4$ **8.** $5x^2 - x + 8$

9.
$$4x^2 - 8x$$

10. $6x^2 - 5x + 3$

11.
$$-2x^2 + 4x + 4$$
 12. $3x^2 + 2x + 2$

B.1 Graphing $f(x) = ax^2$. For use with Exploration 8.1

Essential Question What are some of the characteristics of the graph of a quadratic function of the form $f(x) = ax^2$?

EXPLORATION: Graphing Quadratic Functions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Graph each quadratic function. Compare each graph to the graph of $f(x) = x^2$.

a.
$$g(x) = 3x^2$$



c.
$$g(x) = -0.2x^2$$



b.
$$g(x) = -5x^2$$



d.
$$g(x) = \frac{1}{10}x^2$$



8.1 Graphing $f(x) = ax^2$ (continued)

Communicate Your Answer

2. What are some of the characteristics of the graph of a quadratic function of the form $f(x) = ax^2$?

3. How does the value of *a* affect the graph of $f(x) = ax^2$? Consider 0 < a < 1, a > 1, -1 < a < 0, and a < -1. Use a graphing calculator to verify your answers.

4. The figure shows the graph of a quadratic function of the form $y = ax^2$. Which of the intervals in Question 3 describes the value of *a*? Explain your reasoning.





Core Concepts

Characteristics of Quadratic Functions

The *parent quadratic function* is $f(x) = x^2$. The graphs of all other quadratic functions are *transformations* of the graph of the parent quadratic function.

The lowest point on a parabola that opens up or the highest point on a parabola that opens down is the **vertex.** The vertex of the graph of $f(x) = x^2$ is (0, 0).



The vertical line that divides the parabola into two symmetric parts is the **axis of symmetry.** The axis of symmetry passes through the vertex. For the graph of $f(x) = x^2$, the axis of symmetry is the *y*-axis, or x = 0.

Notes:

Graphing $f(x) = ax^2$ When a > 0

- When 0 < a < 1, the graph of $f(x) = ax^2$ is a vertical shrink of the graph of $f(x) = x^2$.
- When a > 1, the graph of f(x) = ax² is a vertical stretch of the graph of f(x) = x².



Graphing $f(x) = ax^2$ When a < 0

- When -1 < a < 0, the graph of $f(x) = ax^2$ is a vertical shrink with a reflection in the *x*-axis of the graph of $f(x) = x^2$.
- When a < -1, the graph of $f(x) = ax^2$ is a vertical stretch with a reflection in the *x*-axis of the graph of $f(x) = x^2$.



Notes:

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8.1 Practice (continued)

Worked-Out Examples

Example #1

Graph the function. Compare the graph to the graph of $f(x) = x^2$.

 $h(x) = \frac{1}{4}x^2$

x	-4	-2	0	2	4
h(x)	4	1	0	1	4

Both graphs open up and have the same vertex, (0, 0), and the same axis of symmetry, x = 0, but the graph of *h* is wider than the graph of *f*. So, the graph of *h* is a vertical shrink by a factor of $\frac{1}{4}$ of the graph of *f*.

Example #2

Graph the function. Compare the graph to the graph of $f(x) = x^2$.

 $m(x) = -2x^2$

x	-2	-1	0	1	2
m(x)	-8	-2	0	-2	-8

The graphs have the same vertex, (0, 0), and the same axis of symmetry, x = 0, but the graph of *m* opens down and is narrower than the graph of *f*. So, the graph of *m* is a vertical stretch by a factor of 2 and a reflection in the *x*-axis of the graph of *f*.

Practice A

In Exercises 1 and 2, identify characteristics of the quadratic function and its graph.



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 $h(x) = \frac{1}{4}x^2$

2468x



-8-6-4-2



8.1 **Practice** (continued)





In Exercises 9 and 10, determine whether the statement is *always, sometimes,* or *never* true. Explain your reasoning.

- **9.** The graph of $g(x) = ax^2$ is wider than the graph of $f(x) = x^2$ when a > 0.
- **10.** The graph of $g(x) = ax^2$ is narrower than the graph of $f(x) = x^2$ when |a| < 1.

Practice B

In Exercises 1–6, graph the function. Compare the graph to the graph of $f(x) = x^2$.

- **1.** $g(x) = 7x^2$ **2.** $h(x) = 0.25x^2$ **3.** $j(x) = \frac{7}{2}x^2$
- **4.** $g(x) = -\frac{5}{3}x^2$ **5.** $k(x) = -\frac{3}{4}x^2$ **6.** $n(x) = -0.4x^2$
- 7. Describe and correct the error in graphing and comparing $y = x^2$ and $y = -2x^2$.



The graphs have the same vertex and the same axis of symmetry. The graph of $y = -2x^2$ is a reflection in the *x*-axis of the graph of $y = x^2$.

- 8. The arch support of a bridge can be modeled by $y = -\frac{1}{300}x^2$, where x and y are measured in feet.
 - **a.** The width of the arch is 900 feet. Describe the domain of the function. Explain.
 - **b.** Graph the function using the domain in part (a). Find the height of the arch.
- **9.** A parabola opens down and passes through the points (-3, 4) and (1, -2). How do you know that (-3, 4) could be the vertex?
- **10.** Given the parabola $f(x) = ax^2$.
 - **a.** Find the value of a when the graph passes through (3, -1) and a < 0.
 - **b.** Find the value of a when the graph passes through (3, -1) and a > 0. Explain.