

7.6

Factoring Special Products

For use with Exploration 7.6

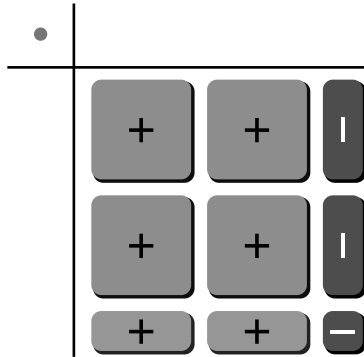
Essential Question How can you recognize and factor special products?

1 EXPLORATION: Factoring Special Products

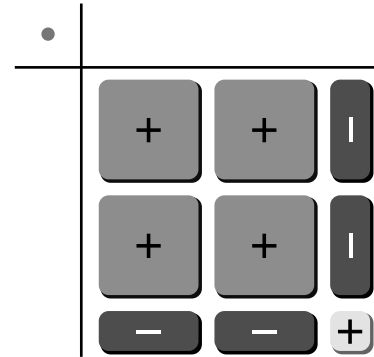
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying. State whether the product is a “special product” that you studied in Section 2.3.

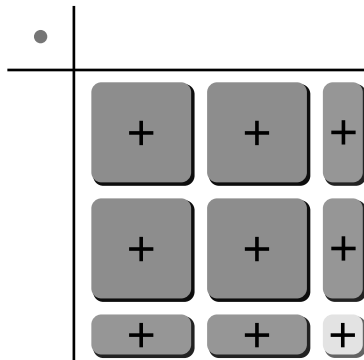
a. $4x^2 - 1 =$ _____



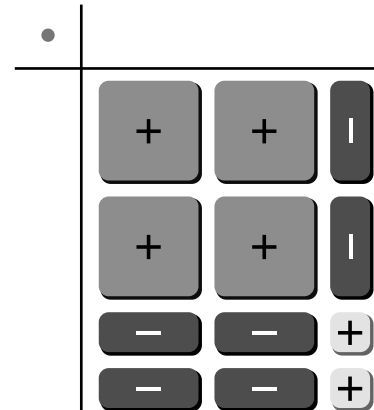
b. $4x^2 - 4x + 1 =$ _____



c. $4x^2 + 4x + 1 =$ _____



d. $4x^2 - 6x + 2 =$ _____

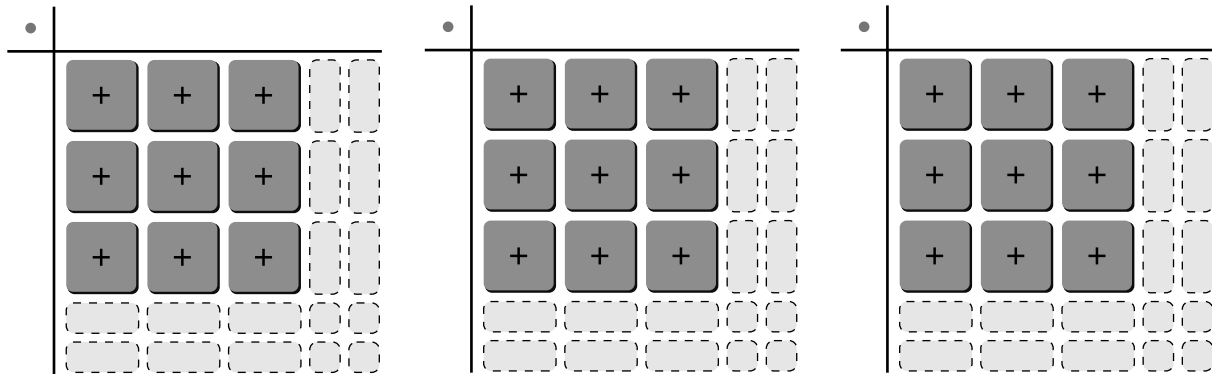


7.6 Factoring Special Products (continued)

2 EXPLORATION: Factoring Special Products

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use algebra tiles to complete the rectangular arrays in three different ways, so that each way represents a different special product. Write each special product in standard form and in factored form.



Communicate Your Answer

3. How can you recognize and factor special products? Describe a strategy for recognizing which polynomials can be factored as special products.

4. Use the strategy you described in Question 3 to factor each polynomial.

a. $25x^2 + 10x + 1$

b. $25x^2 - 10x + 1$

c. $25x^2 - 1$

7.6**Practice**

For use after Lesson 7.6

Core Concepts**Difference of Two Squares Pattern****Algebra**

$$a^2 - b^2 = (a + b)(a - b)$$

Notes:**Example**

$$x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$$

Perfect Square Trinomial Pattern**Algebra**

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Notes:**Example**

$$\begin{aligned} x^2 + 6x + 9 &= x^2 + 2(x)(3) + 3^2 \\ &= (x + 3)^2 \end{aligned}$$

$$\begin{aligned} x^2 - 6x + 9 &= x^2 - 2(x)(3) + 3^2 \\ &= (x - 3)^2 \end{aligned}$$

Worked-Out Examples**Example #1****Factor the polynomial.**

$$\begin{aligned} 16x^2 - 169y^2 &= (4x)^2 - (13y)^2 \\ &= (4x + 13y)(4x - 13y) \end{aligned}$$

$$\text{So, } 16x^2 - 169y^2 = (4x + 13y)(4x - 13y).$$

Example #2**Factor the polynomial.**

$$\begin{aligned} 3z^2 - 27 &= 3(z^2 - 9) \\ &= 3(z^2 - 3^2) \\ &= 3(z + 3)(z - 3) \end{aligned}$$

$$\text{So, } 3z^2 - 27 = 3(z + 3)(z - 3).$$

7.6 Practice (continued)**Practice A**

In Exercises 1–6, factor the polynomial.

1. $s^2 - 49$

2. $t^2 - 81$

3. $16 - x^2$

4. $4g^2 - 25$

5. $36h^2 - 121$

6. $81 - 49k^2$

In Exercises 7–12, use a special product pattern to evaluate the expression.

7. $57^2 - 53^2$

8. $38^2 - 32^2$

9. $68^2 - 64^2$

10. $45^2 - 40^2$

11. $79^2 - 71^2$

12. $86^2 - 84^2$

7.6 Practice (continued)**In Exercises 13–18, factor the polynomial.**

13. $x^2 + 16x + 64$

14. $p^2 + 28p + 196$

15. $r^2 - 26r + 169$

16. $a^2 - 18a + 81$

17. $36c^2 + 84c + 49$

18. $100x^2 - 20x + 1$

In Exercises 19–24, solve the equation.

19. $x^2 - 144 = 0$

20. $9y^2 = 49$

21. $c^2 + 14c + 49 = 0$

22. $d^2 - 4d + 4 = 0$

23. $n^2 + \frac{2}{3}n = -\frac{1}{9}$

24. $-\frac{6}{5}k + \frac{9}{25} = -k^2$

25. The dimensions of a rectangular prism are $(x + 1)$ feet by $(x + 2)$ feet by 4 feet. The volume of the prism is $(24x - 1)$ cubic feet. What is the value of x ?

Practice B

In Exercises 1–3, factor the polynomial.

1. $100 - 49x^2$

2. $121s^2 - 25t^2$

3. $x^2 - 144y^2$

In Exercises 4–6, use a special product pattern to evaluate the expression.

4. $86^2 - 84^2$

5. $44^2 - 39^2$

6. $28^2 - 27^2$

In Exercises 7–9, factor the polynomial.

7. $z^2 + 26z + 169$

8. $16x^2 - 40x + 25$

9. $81a^2 + 36a + 4$

10. The area (in square inches) of a square table can be represented by $25x^2 + 40x + 16$.

- Write an expression that represents the side length of the table.
- Will a square table cloth with side length 60 inches cover the table when $x = 12$?

In Exercises 11–14, solve the equation.

11. $100x^2 = 81$

12. $w^2 + 24w + 144 = 0$

13. $s^2 + 81 = 18s$

14. $y^2 - \frac{1}{3}y = -\frac{1}{36}$

In Exercises 15–17, factor the polynomial.

15. $8y^2 - 72$

16. $7p^2 + 56p + 112$

17. $48t^2 - 72t + 27$

18. The function $y = -16t^2 + 24t$ represents the height y (in feet) of a tennis ball bouncing straight up from the ground t seconds after it bounces. After how many seconds does the tennis ball return to the ground?

19. Tell whether the polynomial can be factored. If not, change the constant term so that the polynomial is a perfect square trinomial.

a. $q^2 + \frac{1}{2}q + \frac{1}{3}$

b. $4x^2 + 28x + 47$

20. A square picture frame has side length x inches. The square opening for a picture within the frame has side length 6 inches.

- Write a polynomial that represents the area of the picture frame, not including the picture.
- The area in part (a) is 64 square inches. What is the side length of the picture frame? Explain your reasoning.