

# 7.5

## Factoring $ax^2 + bx + c$

For use with Exploration 7.5

**Essential Question** How can you use algebra tiles to factor the trinomial  $ax^2 + bx + c$  into the product of two binomials?

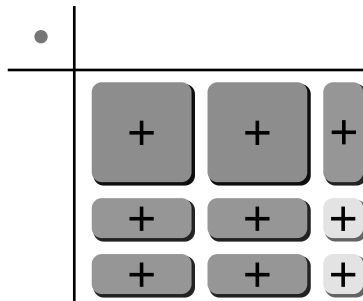
**1 EXPLORATION: Finding Binomial Factors**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

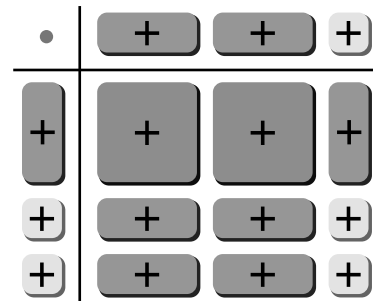
**Work with a partner.** Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

**Sample**  $2x^2 + 5x + 2$

**Step 1** Arrange algebra tiles that model  $2x^2 + 5x + 2$  into a rectangular array.



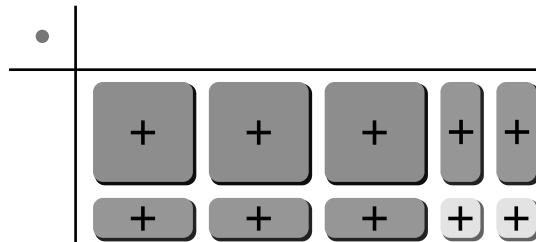
**Step 2** Use additional algebra tiles to model the dimensions of the rectangle.



**Step 3** Write the polynomial in factored form using the dimensions of the rectangle.

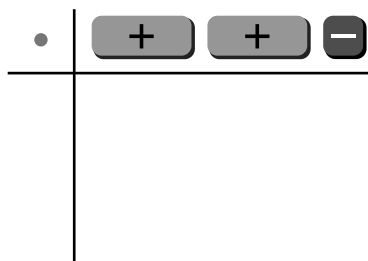
$$\text{Area} = 2x^2 + 5x + 2 = \overset{\text{width}}{\underbrace{(x + 2)}} \overset{\text{length}}{\underbrace{(2x + 1)}}$$

a.  $3x^2 + 5x + 2 =$  \_\_\_\_\_

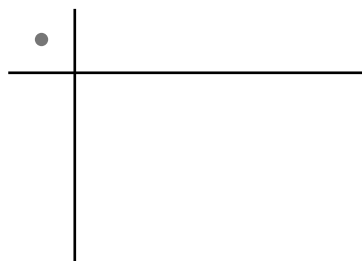


**7.5** Factoring  $ax^2 + bx + c$  (continued)**1** **EXPLORATION:** Finding Binomial Factors (continued)

b.  $4x^2 + 4x - 3 =$  \_\_\_\_\_



c.  $2x^2 - 11x + 5 =$  \_\_\_\_\_

**Communicate Your Answer**

- How can you use algebra tiles to factor the trinomial  $ax^2 + bx + c$  into the product of two binomials?
- Is it possible to factor the trinomial  $2x^2 + 2x + 1$ ? Explain your reasoning.

**7.5****Practice**

For use after Lesson 7.5

Notes:

**Worked-Out Examples****Example #1**

Factor the polynomial.

$$\begin{aligned}9r^2 - 36r - 45 &= 9(r^2 - 4r - 5) \\ &= 9(r + 1)(r - 5)\end{aligned}$$

<b>Factors of -5</b>	1, -5	-1, 5
<b>Sum of factors</b>	-4	4

$$\text{So, } 9r^2 - 36r - 45 = 9(r + 1)(r - 5).$$

**7.5 Practice (continued)****Example #2****Factor the polynomial.**

$$18v^2 - 15v - 18 = 3(6v^2 - 5v - 6)$$

Factors of 6	Factors of -6	Possible factorization	Middle term	
1, 6	1, -6	$(v + 1)$ $(6v - 6)$	$-6v + 6v = 0$	✗
1, 6	6, -1	$(v + 6)$ $(6v - 1)$	$-v + 36v = 35v$	✗
1, 6	-1, 6	$(v - 1)$ $(6v + 6)$	$6v - 6v = 0$	✗
1, 6	-6, 1	$(v - 6)$ $(6v + 1)$	$v - 36v = -35v$	✗
1, 6	2, -3	$(v + 2)$ $(6v - 3)$	$-3v + 12v = 9v$	✗
1, 6	3, -2	$(v + 3)$ $(6v - 2)$	$-2v + 18v = 16v$	✗
1, 6	-2, 3	$(v - 2)$ $(6v + 3)$	$3v - 12v = -9v$	✗
1, 6	-3, 2	$(v - 3)$ $(6v + 2)$	$2v - 18v = -16v$	✗
2, 3	1, -6	$(2v + 1)$ $(3v - 6)$	$-12v + 3v = -9v$	✗
2, 3	6, -1	$(2v + 6)$ $(3v - 1)$	$-2v + 18v = 16v$	✗
2, 3	-1, 6	$(2v - 1)$ $(3v + 6)$	$12v - 3v = 9v$	✗
2, 3	-6, 1	$(2v - 6)$ $(3v + 1)$	$2v - 18v = -16v$	✗
2, 3	2, -3	$(2v + 2)$ $(3v - 3)$	$-6v + 6v = 0$	✗
2, 3	3, -2	$(2v + 3)$ $(3v - 2)$	$-4v + 9v = 5v$	✗
2, 3	-2, 3	$(2v - 2)$ $(3v + 3)$	$6v - 6v = 0$	✗
2, 3	-3, 2	$(2v - 3)$ $(3v + 2)$	$4v - 9v = -5v$	✓

$$\text{So, } 18v^2 - 15v - 18 = 3(2v - 3)(3v + 2).$$

**7.5 Practice (continued)****Practice A**

In Exercises 1–18, factor the polynomial.

1.  $2c^2 - 14c - 36$

2.  $4a^2 + 8a - 140$

3.  $3x^2 - 6x - 24$

4.  $2d^2 - 2d - 60$

5.  $5s^2 + 55s + 50$

6.  $3q^2 + 30q + 27$

7.  $12g^2 - 37g + 28$

8.  $6k^2 - 11k + 4$

9.  $9w^2 + 9w + 2$

10.  $12a^2 + 5a - 2$

11.  $15b^2 + 14b - 8$

12.  $5t^2 + 12t - 9$

13.  $-12b^2 + 5b + 2$

14.  $-6x^2 + x + 15$

15.  $-60g^2 - 11g + 1$

16.  $-2d^2 - d + 6$

17.  $-3r^2 - 4r - 1$

18.  $-8x^2 + 14x - 5$

19. The length of a rectangular shaped park is  $(3x + 5)$  miles. The width is  $(2x + 8)$  miles. The area of the park is 360 square miles. What are the dimensions of the park?
20. The sum of two numbers is 8. The sum of the squares of the two numbers is 34. What are the two numbers?

## Practice B

In Exercises 1–12, factor the polynomial.

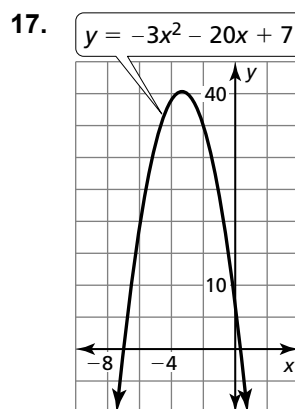
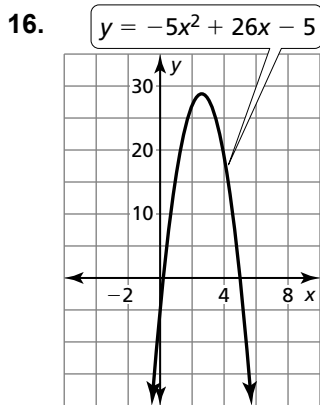
- |                       |                        |                         |
|-----------------------|------------------------|-------------------------|
| 1. $5x^2 - 5x - 30$   | 2. $8x^2 - 16x - 192$  | 3. $6x^2 + 48x + 42$    |
| 4. $2x^2 + 17x - 9$   | 5. $12p^2 - 7p - 10$   | 6. $10w^2 + 24w + 8$    |
| 7. $3y^2 + y - 14$    | 8. $12j^2 - 32j + 5$   | 9. $15d^2 + 16d - 15$   |
| 10. $-9v^2 - 22v - 8$ | 11. $-14m^2 + 13m - 3$ | 12. $-20q^2 + 56q - 15$ |
13. Describe and correct the error in factoring the polynomial.

$\times \quad 6x^2 - 4x + 2 = (2x - 2)(3x + 1)$

In Exercises 14 and 15, solve the equation.

14.  $-12w^2 + 20w - 3 = 0$                       15.  $18t^2 - 2 = 5t$

In Exercises 16 and 17, find the  $x$ -coordinates of the points where the graph crosses the  $x$ -axis.



18. The length of a rectangular patio is 8 feet less than twice its width. The area of the patio is 280 square feet. Find the dimensions of the patio.
19. For what values of  $t$  can  $6x^2 + tx + 25$  be written as the product of two binomials?

In Exercises 20 and 21, factor the polynomial.

20.  $-10r^2 - 11sr + 6s^2$                       21.  $12x^3 + 8x^2y - 20xy^2$