CHAPTER 7 Polynomial Equations and Factoring

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7 Maintaining Mathematical Proficiency

Simplify the expression.

1. 5x - 6 + 3x **2.** 3t + 7 - 3t - 4 **3.** 8s - 4 + 4s - 6 - 5s

4.
$$9m + 3 + m - 3 + 5m$$
 5. $-4 - 3p - 7 - 3p - 4$ **6.** $12(z - 1) + 4$

7.
$$-6(x+2) - 4$$
 8. $3(h+4) - 3(h-4)$ **9.** $7(z+4) - 3(z+2) - 2(z-3)$

| Find the greatest common factor. | | | | | |
|----------------------------------|--------|-----|--------|-----|--------|
| 10. | 24, 32 | 11. | 30, 55 | 12. | 48, 84 |
| | | | | | |
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| | | | | | |
| | | | | | |

14. 42,60

16. Explain how to find the greatest common factor of 42, 70, and 84.

13. 28, 72

15. 35,99

1

Adding and Subtracting Polynomials 7.1 For use with Exploration 7.1 Essential Question How can you add and subtract polynomials? **EXPLORATION:** Adding Polynomials

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Write the expression modeled by the algebra tiles in each step.



2 **EXPLORATION:** Subtracting Polynomials

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Write the expression modeled by the algebra tiles in each step.

| Step 1 + + + + + - + \bigcirc | $(x^2 + 2x + 2) - (x - 1)$ |
|--|----------------------------|
| Step 2 + + + + + + + + + + + + + + + + + + | |
| Step 3 + + + + + + + + + | |

7.1 Adding and Subtracting Polynomials (continued)



Communicate Your Answer

3. How can you add and subtract polynomials?

4. Use your methods in Question 3 to find each sum or difference.

a.
$$(x^2 + 2x - 1) + (2x^2 - 2x + 1)$$

b. $(4x + 3) + (x - 2)$

c.
$$(x^2 + 2) - (3x^2 + 2x + 5)$$

d. $(2x - 3x) - (x^2 - 2x + 4)$



Notes:

Core Concepts

Polynomials

A **polynomial** is a monomial or a sum of monomials. Each monomial is called a *term* of the polynomial. A polynomial with two terms is a **binomial**. A polynomial with three terms is a **trinomial**.

| Binomial Trinomia | |
|-------------------|----------------|
| 5x + 2 | $x^2 + 5x + 2$ |

The **degree of a polynomial** is the greatest degree of its terms. A polynomial in one variable is in **standard form** when the exponents of the terms decrease from left to right. When you write a polynomial in standard form, the coefficient of the first term is the **leading coefficient**.



Notes:

Worked-Out Examples

Example #1

Write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

 $8d - 2 - 4d^3$

You can write the polynomial $8d - 2 - 4d^3$ in standard form as $-4d^3 + 8d - 2$.

The greatest degree is 3, so the degree of the polynomial 3.

The leading coefficient is -4.

The polynomial has 3 terms, so it is a trinomial.

7.1 Practice (continued)

Example #2

Find the difference.

 $(4m^{2} - m + 2) - (-3m^{2} + 10m + 4)$ = $4m^{2} - m + 2 + 3m^{2} - 10m - 4$ = $(4m^{2} + 3m^{2}) + (-m - 10m) + (2 - 4)$ = $7m^{2} - 11m - 2$ Alternate solution: $4m^{2} - m + 2 - 4m^{2} - m + 2$

 $\frac{-(-3m^2+10m+4)}{7m^2-11m-2} \Rightarrow \frac{3m^2-10m-4}{7m^2-11m-2}$

Practice A

In Exercises 1–8, find the degree of the monomial.

| 5. | $7x^2y$ | 6. $4r^2s^3t$ | 7. $10mn^3$ | 8. $\frac{2}{3}$ |
|----|---------|----------------------|--------------------|------------------|
|----|---------|----------------------|--------------------|------------------|

7.1 Practice (continued)

In Exercises 9–12, write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

9.
$$x + 3x^2 + 5$$
 10. $\sqrt{5} y$ **11.** $3x^5 + 6x^8$ **12.** $f^2 - 2f + f^4$

In Exercises 13–16, find the sum.

13.
$$(-4x + 9) + (6x - 14)$$
 14. $(-3a - 2) + (7a + 5)$

15.
$$(x^2 + 3x + 5) + (-x^2 + 6x - 4)$$
 16. $(t^2 + 3t^3 - 3) + (2t^2 + 7t - 2t^3)$

In Exercises 17–20, find the difference.

17.
$$(g-4) - (3g-6)$$
 18. $(-5h-2) - (7h+6)$

19.
$$(-x^2 - 5) - (-3x^2 - x - 8)$$

20. $(k^2 + 6k^3 - 4) - (5k^3 + 7k - 3k^2)$

Practice B

In Exercises 1–3, find the degree of the monomial.

1.
$$-3.25n^8$$
 2. $\frac{1}{5}x^4yz^2$ **3.** uv^3w^9

In Exercises 4–6, write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

4.
$$3t - 8t^2 + 10t^5$$
 5. $\frac{2}{9}n^2 - \pi n + 3n^4$ **6.** $\sqrt{14}p^5$

7. The monthly profit for a small company is represented by $250x^5 - 42x^2 + 112x$, where x is the number of beds sold. Classify the polynomial by the number of terms. What is its degree?

In Exercises 8–11, find the sum.

8. $(-2t^2 - 7t + 5) + (-8t^2 + 4t - 3)$ **9.** $(8y^2 - 2y + 4) + (5y^2 - 7y)$ **10.** $(3k - 5k^3 + 9) + (8k^3 - 4k + 8)$ **11.** $(3q^2 - 7q - 6) + (2q^2 - 5q^3 + 8q)$

In Exercises 12–15, find the difference.

12.
$$(t^3 - 5t^2 - 7) - (t - 11)$$

13. $(-w - 13) - (-3w^3 + w^2 + 6w)$
14. $(x^4 - x^2 + 9) - (13 - 6x^2 + 8x)$
15. $(3g - 5g^3 + 6g^2) - (12g^3 + 9g - 10)$

16. The number of economy-size cars rented in w weeks is represented by 152 + 3w. The number of full-size cars rented in w weeks is represented by 99 + 2w. Write a polynomial that represents how many more economy cars are rented in w weeks than full-size cars.

In Exercises 17 and 18, find the sum or difference.

17.
$$(g^2 - 9h^2) + (g^2 - 15gh + 8h^2)$$
 18. $(-m^2 - 5mn) - (m^2 + 3mn - 9n^2)$

19. The polynomial $-16t^2 + v_0t + s_0$ represents the height (in feet) of an object, where v_0 is the initial vertical velocity (in feet per second), s_0 is the initial height of the object (in feet), and t is the time (in seconds). Write a polynomial that represents the height of an object that has initial velocity 25 feet per second and initial height 4 feet. Then find the height of the object after 1 second.