

# 6.7

## Geometric Sequences

For use with Exploration 6.7

**Essential Question** How can you use a geometric sequence to describe a pattern?

In a **geometric sequence**, the ratio between each pair of consecutive terms is the same. This ratio is called the **common ratio**.

**1 EXPLORATION:** Describing Calculator Patterns

**Work with a partner.** Enter the keystrokes on a calculator and record the results in the table. Describe the pattern.

a. Step 1     **2** **=**

Step 2   **×** **2** **=**

Step 3   **×** **2** **=**

Step 4   **×** **2** **=**

Step 5   **×** **2** **=**

<b>Step</b>	1	2	3	4	5
<b>Calculator display</b>					

b. Step 1     **6** **4** **=**

Step 2   **×** **.** **5** **=**

Step 3   **×** **.** **5** **=**

Step 4   **×** **.** **5** **=**

Step 5   **×** **.** **5** **=**

<b>Step</b>	1	2	3	4	5
<b>Calculator display</b>					

c. Use a calculator to make your own sequence. Start with any number and multiply by 3 each time. Record your results in the table.

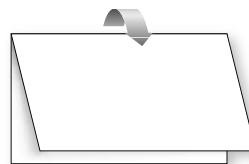
<b>Step</b>	1	2	3	4	5
<b>Calculator display</b>					

d. Part (a) involves a geometric sequence with a common ratio of 2. What is the common ratio in part (b)? part (c)?

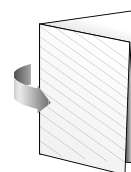
**6.7 Geometric Sequences (continued)****2 EXPLORATION:** Folding a Sheet of Paper

**Work with a partner.** A sheet of paper is about 0.1 millimeter thick.

- a. How thick will it be when you fold it in half once? twice? three times?



- b. What is the greatest number of times you can fold a piece of paper in half? How thick is the result?



- c. Do you agree with the statement below? Explain your reasoning.

*“If it were possible to fold the paper in half 15 times, it would be taller than you.”*

**Communicate Your Answer**

3. How can you use a geometric sequence to describe a pattern?
4. Give an example of a geometric sequence from real life other than paper folding.

# 6.7

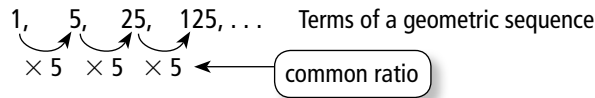
## Practice

For use after Lesson 6.7

### Core Concepts

#### Geometric Sequence

In a **geometric sequence**, the ratio between each pair of consecutive terms is the same. This ratio is called the **common ratio**. Each term is found by multiplying the previous term by the common ratio.



#### Notes:

#### Equation for a Geometric Sequence

Let  $a_n$  be the  $n$ th term of a geometric sequence with first term  $a_1$  and common ratio  $r$ . The  $n$ th term is given by

$$a_n = a_1 r^{n-1}.$$

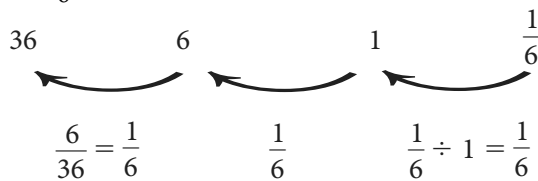
#### Notes:

### Worked-Out Examples

#### Example #1

Find the common ratio of the geometric sequence.

36, 6, 1,  $\frac{1}{6}$ , ...



The common ratio is  $\frac{1}{6}$ .

#### Example #2

Write an equation for the  $n$ th term of the geometric sequence. Then find  $a_6$ .

2, 8, 32, 128, ...

$$\begin{aligned} a_1 = 2, r = \frac{8}{2} = 4 & \quad a_n = 2(4)^{n-1} \\ a_n = a_1 r^{n-1} & \quad a_6 = 2(4)^{6-1} \\ a_n = 2(4)^{n-1} & \quad = 2(4)^5 \\ & \quad = 2(1024) \\ & \quad = 2048 \end{aligned}$$

The 6th term of the geometric sequence is 2048.

**6.7 Practice (continued)**

**Practice A**

In Exercises 1–6, determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

1. 1, -4, 16, -64, ...      2. 3, 7, 11, 15, ...      3. 2, 4, 8, 32, ...

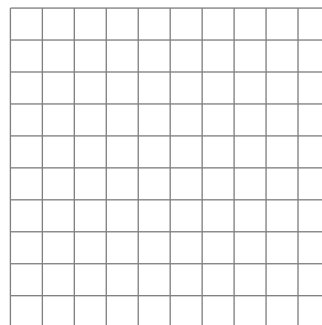
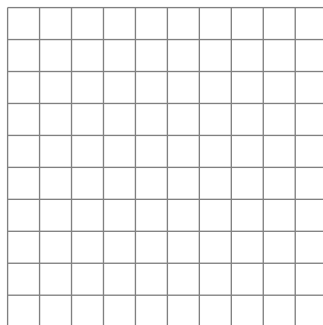
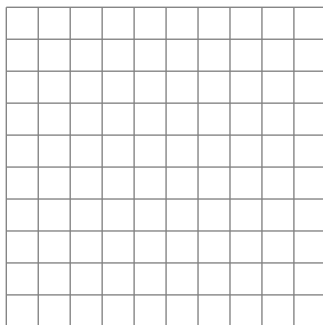
4. 12, 9, 7, 5, ...      5. 6, 18, 54, 162, ...      6. 11, 19, 27, 35, ...

In Exercises 7–9, write the next three terms of the geometric sequence.

7. 7, 21, 63, 189, ...      8. 576, 288, 144, 72, ...      9. 5, -10, 20, -40, ...

In Exercises 10–12, write the next three terms of the geometric sequence. Then graph the sequence.

10. 12, 6, 3,  $\frac{3}{2}$ , ...      11. 3, 12, 48, 192, ...      12. 0.008, 0.04, 0.2, 1, ...



**6.7 Practice (continued)**

In Exercises 13–20, write an equation for the  $n$ th term of the geometric sequence. Then find  $a_6$ .

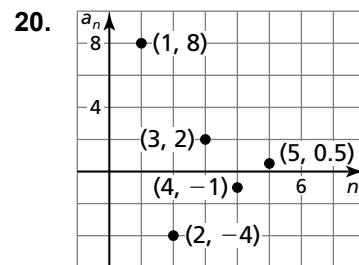
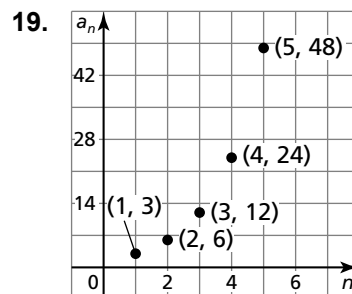
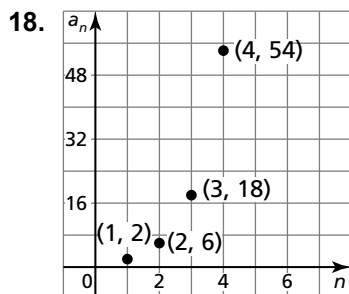
13. 6561, 2187, 729, 243, ...    14. 8, -24, 72, -216, ...    15. 3, 15, 75, 375, ...

16.

$n$	1	2	3	4
$a_n$	2916	972	324	108

17.

$n$	1	2	3	4
$a_n$	11	44	176	704



## Practice B

In Exercises 1–3, find the common ratio of the geometric sequence.

1. 5, 20, 80, 320, ...      2. 144, -72, 36, -18, ...      3. 24, 84, 294, 1029, ...

In Exercises 4–7, determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

4. 2.786, 27.86, 278.6, 2786, ...      5. 86, 71, 56, 41, ...  
6. 4, -10, 16, -28, ...      7. 112, -28, 7,  $-\frac{7}{4}$ , ...

In Exercises 8 and 9, write the next three terms of the geometric sequence. Then graph the sequence.

8. -2, -12, -72, -432, ...      9.  $\frac{54}{25}$ ,  $\frac{18}{5}$ , 6, 10, ...

In Exercises 10–13, write an equation for the  $n$ th term of the geometric sequence. Then find  $a_6$ .

10.  $\frac{3}{125}$ ,  $\frac{3}{25}$ ,  $\frac{3}{5}$ , 3, ...      11. 0.2, 1.6, 12.8, 102.4, ...

12.

$n$	1	2	3	4
$a_n$	2436	-243.6	24.36	-2.436

13.

$n$	1	2	3	4
$a_n$	-1458	-162	-18	-2

14. An archery competition begins with 256 competitors. After the first round, one-fourth of the competing group remains. After the second round, one-fourth of the now smaller competing group remains. The last round is when there are fewer than five members in the competing group.

- a. Which round is the last round?  
b. How many competitors are in the last round?

15. What is the 10th term of the geometric sequence where  $a_3 = \frac{8}{3}$  and  $r = \frac{2}{3}$ ?

16. Find the sum of the terms of the geometric sequence

$$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$$

Explain your reasoning.