

6.4**Comparing Linear and Exponential Functions**
For use with Exploration 6.4

Essential Question How can you compare the growth rates of linear and exponential functions?

1 EXPLORATION: Comparing Values

Work with a partner. An art collector buys two paintings. The value of each painting after t year is y dollars. Complete each table. Compare the values of the two paintings. Which painting's value has a constant growth rate? Which painting's value has an increasing growth rate? Explain your reasoning.

t	$y = 19t + 5$
0	
1	
2	
3	
4	

t	$y = 3^t$
0	
1	
2	
3	
4	

6.4 Comparing Linear and Exponential Functions (continued)**2 EXPLORATION: Comparing Values**

Work with a partner. Analyze the values of the two paintings over the given time periods. The value of each painting after t years is y dollars. Which painting's value eventually overtakes the other?

t	$y = 19t + 5$
4	
5	
6	
7	
8	
9	

t	$y = 3^t$
4	
5	
6	
7	
8	
9	

3 EXPLORATION: Comparing Graphs

Work with a partner. Use the tables in Explorations 1 and 2 to graph $y = 19t + 5$ and $y = 3^t$ in the same coordinate plane. Compare the graphs of the functions.

Communicate Your Answer

- How can you compare the growth rates of linear and exponential functions?
- Which function has a growth rate that is eventually much greater than the growth rates of the other function? Explain your reasoning.

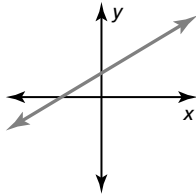
6.4**Practice**

For use after Lesson 6.4

Core Concepts**Linear, Exponential, and Quadratic Functions**

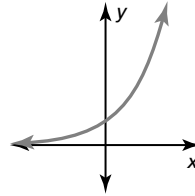
Linear Function

$$y = mx + b$$



Exponential Function

$$y = ab^x$$

**Notes:****Differences and Ratios of Functions**

You can use patterns between consecutive data pairs to determine which type of function models the data.

- **Linear Function** The differences of consecutive y -values are constant.
- **Exponential Function** Consecutive y -values have a common *ratio*.

In each case, the differences of consecutive x -values need to be constant.

Notes:**Comparing Functions Using Average Rates of Change**

As a and b increase, the average rate of change between $x = a$ and $x = b$ of an increasing exponential function $y = f(x)$ will eventually exceed the average rate of change between $x = a$ and $x = b$ of an increasing linear function $y = g(x)$. So, as x increases, $f(x)$ will eventually exceed $g(x)$.

Notes:

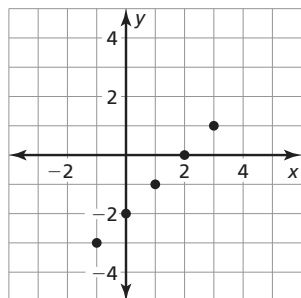
6.4 Practice (continued)

Worked-Out Examples

Example #1

Plot the points. Tell whether the points appear to represent a linear function, an exponential function, or neither.

$(-1, -3), (0, -2), (1, -1), (2, 0), (3, 1)$



The points appear to lie on a straight line. So, they appear to represent a linear function.

Example #2

Tell whether the table of values represents a linear or an exponential function. Then write the function.

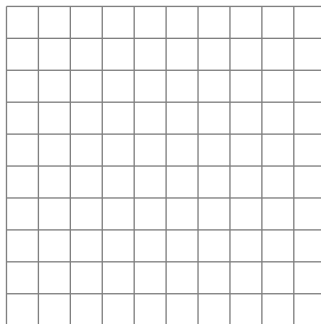
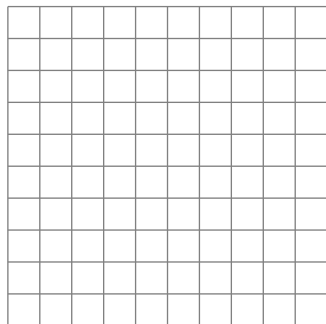
		+1	+1	+1	+1
<i>x</i>	-2	-1	0	1	2
<i>y</i>	$\frac{1}{5}$	1	5	25	125

Consecutive *y*-values have a common ratio of 5, and the *y*-intercept is 5. So, the table represents the exponential function $y = 5(5)^x$.

Practice A

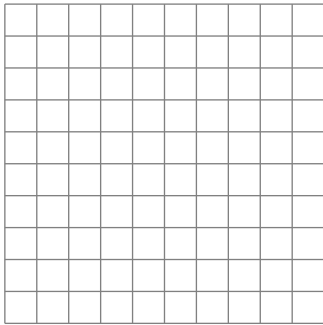
In Exercises 1–4, plot the points. Tell whether the points appear to represent a *linear function*, an *exponential function*, or *neither*.

1. $(-3, 2), (-2, 4), (-4, 4), (-1, 8), (-5, 8)$ 2. $(-3, 1), (-2, 2), (-1, 4), (0, 8), (2, 14)$

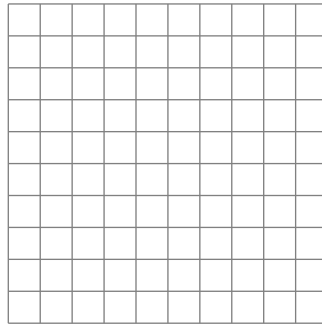


6.4 Practice (continued)

3. $(4, 0), (2, 1), (0, 3), (-1, 6), (-2, 10)$



4. $(2, -4), (0, -2), (-2, 0), (-4, 2), (-6, 4)$



In Exercises 5 and 6, tell whether the table of values represents a *linear*, or an *exponential* function.

5.

x	-2	-1	0	1	2
y	7	4	1	-2	-5

6.

x	-2	-1	0	1	2
y	$\frac{1}{18}$	$\frac{1}{3}$	2	12	72

In Exercises 7 and 8, tell whether the data represent a *linear*, or an *exponential* function. Then write the function.

7. $(-2, -4), (-1, -1), (0, 2), (1, 5), (2, 8)$

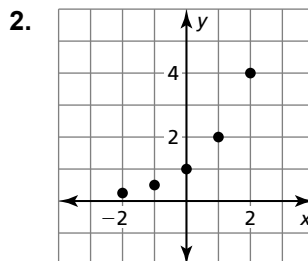
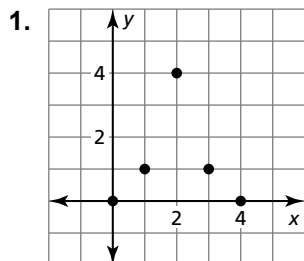
8. $(-2, 1.75), (-1, 3.5), (0, 7), (1, 14), (2, 28)$

9. A person invests \$1000 into an account that earns compound interest. The table shows the amount A (in dollars) in the account after t (in years) time has passed. Tell whether the data can be modeled by a *linear* or an *exponential* function. Explain.

Time, t	0	1	2	3	4
Amount, A	1000	1050	1102.50	1157.63	1215.51

Practice B

In Exercises 1 and 2, tell whether the points appear to represent a *linear function*, an *exponential function*, or *neither*.



In Exercises 3–6, plot the points. Tell whether the points appear to represent a *linear function*, an *exponential function*, or *neither*.

3. $(2, \frac{1}{16}), (1, \frac{1}{4}), (0, 1), (-1, 4), (-2, 16)$
4. $(-1, 5), (0, 0), (1, -1), (2, 0), (3, 5)$
5. $(-4, -3), (-2, -2), (0, -1), (2, 0), (4, 1)$
6. $(-3, -6), (-2, -4), (-1, -2), (0, 0), (1, 2)$

In Exercises 7–10, tell whether the table of values represents a *linear*, or an *exponential function*. Then write the function.

7.

x	-3	-2	-1	0	1	2
y	8	4	2	1	0.5	0.25

8.

x	1	2	3	4	5	6
y	2	0	-2	-4	-6	-8

9.

x	1	2	3	4	5	6
y	-8	-5	-2	1	4	7

10.

x	-1	0	1	2	3
y	3	$\frac{3}{2}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{16}$

11. Write a function that has an average rate of change that is constant.