

**5.2****Solving Systems of Linear Equations by Substitution**

For use with Exploration 5.2

**Essential Question** How can you use substitution to solve a system of linear equations?

**1 EXPLORATION:** Using Substitution to Solve Systems

**Work with a partner.** Solve each system of linear equations using two methods.

**Method 1** Solve for  $x$  first.

Solve for  $x$  in one of the equations. Substitute the expression for  $x$  into the other equation to find  $y$ . Then substitute the value of  $y$  into one of the original equations to find  $x$ .

**Method 2** Solve for  $y$  first.

Solve for  $y$  in one of the equations. Substitute the expression for  $y$  into the other equation to find  $x$ . Then substitute the value of  $x$  into one of the original equations to find  $y$ .

Is the solution the same using both methods? Explain which method you would prefer to use for each system.

a.  $x + y = -7$   
 $-5x + y = 5$

b.  $x - 6y = -11$   
 $3x + 2y = 7$

c.  $4x + y = -1$   
 $3x - 5y = -18$

**5.2 Solving Systems of Linear Equations by Substitution (continued)****2 EXPLORATION: Writing and Solving a System of Equations**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

- a. Write a random ordered pair with integer coordinates. One way to do this is to use a graphing calculator. The ordered pair generated at the right is  $(-2, -3)$ .

Choose two  
random integers  
between  $-5$  and  $5$ .

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randInt(-5, 5, 2)
{-2 -3}
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- b. Write a system of linear equations that has your ordered pair as its solution.
- c. Exchange systems with your partner and use one of the methods from Exploration 1 to solve the system. Explain your choice of method.

**Communicate Your Answer**

3. How can you use substitution to solve a system of linear equations?
4. Use one of the methods from Exploration 1 to solve each system of linear equations. Explain your choice of method. Check your solutions.

a.  $x + 2y = -7$   
 $2x - y = -9$

b.  $x - 2y = -6$   
 $2x + y = -2$

c.  $-3x + 2y = -10$   
 $-2x + y = -6$

d.  $3x + 2y = 13$   
 $x - 3y = -3$

e.  $3x - 2y = 9$   
 $-x - 3y = 8$

f.  $3x - y = -6$   
 $4x + 5y = 11$

**5.2****Practice**

For use after Lesson 5.2

**Core Concepts****Solving a System of Linear Equations by Substitution****Step 1** Solve one of the equations for one of the variables.**Step 2** Substitute the expression from Step 1 into the other equation and solve for the other variable.**Step 3** Substitute the value from Step 2 into one of the original equations and solve.**Notes:****Worked-Out Examples****Example #1****Solve the system of linear equations by substitution. Check your solution.**

$$x = 17 - 4y$$

$$y = x - 2$$

Substitute  $17 - 4y$  for  $x$  in Equation 2 and solve for  $y$ .

$$y = x - 2$$

$$y = (17 - 4y) - 2$$

$$y = 17 - 4y - 2$$

$$y = -4y + 15$$

$$\begin{array}{r} +4y \\ \hline 5y = 15 \end{array}$$

$$5y = 15$$

$$\frac{5y}{5} = \frac{15}{5}$$

$$y = 3$$

Substitute 3 for  $y$  in Equation 1 and solve for  $x$ .

$$x = 17 - 4y$$

$$x = 17 - 4(3)$$

$$x = 17 - 12$$

$$x = 5$$

**Check**  $x = 17 - 4y$        $y = x - 2$

$$5 \stackrel{?}{=} 17 - 4(3) \quad 3 \stackrel{?}{=} 5 - 2$$

$$5 \stackrel{?}{=} 17 - 12 \quad 3 = 3 \checkmark$$

$$5 = 5 \checkmark$$

The solution is  $(5, 3)$ .

**5.2 Practice (continued)****Example #2**

Solve the system of linear equations by substitution. Check your solution.

$$11x - 7y = -14$$

$$x - 2y = -4$$

$$\begin{aligned} \text{Step 1} \quad x - 2y &= -4 \\ x - 2y + 2y &= -4 + 2y \\ x &= 2y - 4 \end{aligned}$$

$$\begin{aligned} \text{Step 2} \quad 11x - 7y &= -14 \\ 11(2y - 4) - 7y &= -14 \\ 11(2y) - 11(4) - 7y &= -14 \\ 22y - 44 - 7y &= -14 \\ 15y - 44 &= -14 \\ \quad +44 \quad +44 & \\ \quad 15y &= 30 \\ \quad \frac{15y}{15} &= \frac{30}{15} \\ \quad y &= 2 \end{aligned}$$

$$\begin{aligned} \text{Step 3} \quad x - 2y &= -4 \\ x - 2(2) &= -4 \\ x - 4 &= -4 \\ \quad +4 \quad +4 & \\ \quad x &= 0 \end{aligned}$$

$$\begin{array}{ll} \text{Check} \quad 11x - 7y = -14 & x - 2y = -4 \\ 11(0) - 7(2) \stackrel{?}{=} -14 & 0 - 2(2) \stackrel{?}{=} -4 \\ 0 - 14 \stackrel{?}{=} -14 & 0 - 4 \stackrel{?}{=} -4 \\ -14 = -14 \checkmark & -4 = -4 \checkmark \end{array}$$

The solution is (0, 2).

**Practice A**

In Exercises 1–18, solve the system of linear equations by substitution. Check your solution.

$$\begin{aligned} 1. \quad 2x + 2y &= 10 \\ y &= 5 + x \end{aligned}$$

$$\begin{aligned} 2. \quad 2x - y &= 3 \\ x &= -2y - 1 \end{aligned}$$

$$\begin{aligned} 3. \quad x - 3y &= -1 \\ x &= y \end{aligned}$$

$$\begin{aligned} 4. \quad x - 2y &= -3 \\ y &= x + 1 \end{aligned}$$

$$\begin{aligned} 5. \quad 2x + y &= 3 \\ x &= 3y + 5 \end{aligned}$$

$$\begin{aligned} 6. \quad 3x + y &= -5 \\ y &= 2x + 5 \end{aligned}$$

$$\begin{aligned} 7. \quad y &= 2x + 8 \\ y &= -2x \end{aligned}$$

$$\begin{aligned} 8. \quad y &= \frac{3}{4}x + 1 \\ y &= \frac{1}{4}x + 3 \end{aligned}$$

$$\begin{aligned} 9. \quad 2x - 3y &= 0 \\ y &= 4 \end{aligned}$$

**5.2 Practice (continued)**

10.  $x + y = 3$   
 $2x + 4y = 8$

11.  $y = \frac{1}{2}x + 1$   
 $y = -\frac{1}{2}x + 9$

12.  $3x - 2y = 3$   
 $4x - y = 4$

13.  $7x - 4y = 8$   
 $5x - y = 2$

14.  $y = \frac{3}{5}x - 12$   
 $y = \frac{1}{3}x - 8$

15.  $3x - 4y = -1$   
 $5x + 2y = 7$

16.  $y = -x + 3$   
 $x + 2y = 0$

17.  $y - 5x = -2$   
 $-4x + y = 2$

18.  $4x - 8y = 3$   
 $8x + 4y = 1$

19. An adult ticket to a museum costs \$3 more than a children's ticket. When 200 adult tickets and 100 children's tickets are sold, the total revenue is \$2100. What is the cost of a children's ticket?

## Practice B

In Exercises 1–6, solve the system of linear equations by substitution. Check your solution.

1.  $2x + 2y = 4$   
 $y = 12 - 3x$

2.  $-2x + 9y = 15$   
 $x + 7 = 4$

3.  $x - y = 4$   
 $2x - 3y = 3$

4.  $4x + 3y = -1$   
 $3x + y = -7$

5.  $5x + 5y = -10$   
 $3x - 7y = 4$

6.  $-x + y = 7$   
 $6x - y = -7$

7. A humane society has 73 dogs and cats to be adopted. The number of cats is 10 more than twice the number of dogs. Write a system of linear equations that represents this situation. How many of each animal is up for adoption?

In Exercises 8–10, write a system of linear equations that has the ordered pair as its solution.

8.  $(-6, -2)$

9.  $(-12, 18)$

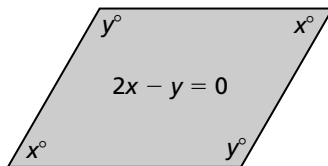
10.  $(2, 0)$

11. A wedding planner purchased both small and large lanterns for a wedding reception. The planner purchased a total of 40 lanterns for a purchase price of \$1180. How many of each size lantern did the planner purchase?

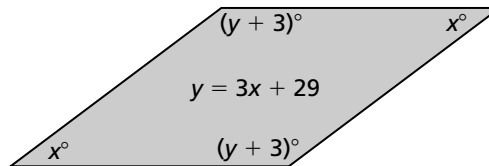
<b>Lantern</b>	Small	Large
<b>Price</b>	\$25	\$40

In Exercises 12 and 13, (a) write an equation that represents the sum of the angle measures of the parallelogram and (b) use your equation and the equation shown to find the values of  $x$  and  $y$ .

12.



13.



14. Write a system of linear equations in which  $(2, -1)$  is a solution of Equation 1 but not a solution of Equation 2, and  $(5, 5)$  is a solution of the system.